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ON THE ELECTRON THEORY OF THERMAL RADIATION FOR SMALL VALUES OF (λT).

By Jakob Kunz.

TWO theories, starting from different assumptions and proceeding in very different ways, account for the laws of thermal radiation of the black body. The elementary system emitting and absorbing heat waves considered by M. Planck is an oscillator consisting of two electrical poles of opposite sign that are movable on a straight line. Making use of the general principles of electrodynamics, thermodynamics and probability, M. Planck finds the following expression for the distribution of energy in the spectrum of radiations of the black body :

$$E_\lambda = \frac{C}{\lambda^5} \frac{e^{-\frac{c}{\lambda T}}}{1 - e^{-\frac{c}{\lambda T}}}$$

Among all the numerous expressions given for this quantity that of Plank alone seems to coincide in a satisfactory way with the experimental data for all values of λ and T. For large values of λT this expression reduces to

$$E_\lambda = C\frac{T}{\lambda^4}$$

a relation that has been suggested by Lord Raleigh. For small values of λT and for visible light Planck's law reduces to

$$E_\lambda = C\frac{e^{-\frac{c}{\lambda T}}}{\lambda^5}$$

due to Wien.

The second theory of radiation, first suggested by J. J. Thomson and then worked out by H. A. Lorentz, considers the radiation of heat and light as of the same nature as Röntgen rays. Röntgen rays are considered as electromagnetic pulses due to the impact of cathode rays against a solid obstacle. In the same way radiation of heat and light may be produced by electrons that come into collision with the atoms of a metal. H. A. Lorentz found

$$E_\lambda = \frac{16}{3}\frac{\pi a T}{\lambda^4}$$

for the amount of energy per unit volume whose wave-length is λ, when a is the gas constant and T the absolute temperature.

For long wave-lengths this expression agrees well with the results of experiments and Lorentz found from the measurements made by Lummer and Pringsheim and Kurlbaum on the amount of radiation given out by hot bodies for $a = 1, 2.10^{-16}$, a value that coincides very closely with the gas constant. We should expect however that most of the energy radiated in consequence of collisions should be contained in the region of short wave-length. An electron coming into collision with an atom emits pulses of intense electric and magnetic force, the thickness of these pulses will be the distance over which light travels during the time occupied by a collision. The disturbance in the electric and magnetic field round about the electron when colliding with an atom consists in a succession of exceedingly thin pulses. Now the part of the energy of radiation calculated by Lorentz corresponds to wave-lengths that are long in comparison with the distance traveled over by light not in the short interval of a collision but in the much longer interval which elapses between two collisions. Lorentz's theory does not explain why there is a maximum of energy of radiation in the spectrum of the black body nor why this maximum moves towards the blue end with increasing temperature. The theory does not account for the fact that a body gets red hot and white hot when its temperature is sufficiently increased. J. J. Thomson in a very suggestive investigation of the electrical origin of the radiation from hot bodies[1] has given a very lucid deduction and extension

[1] Phil. Mag., sixth series, Vol. XIV., p. 217, 1907.

of the results obtained by Lorentz. According to this investigation the character of the radiation depends upon the time of a collision between the electron and an atom. If the time of collision is so prolonged that light during this time can travel over a distance comparable with the wave-length of visible light, then the resulting radiation will be visible light and the maximum intensity of the light will be in that part of the spectrum where the wave-length is comparable with the distance traveled by an electromagnetic wave during a collision. The period of vibration of the light of maximum intensity is comparable with the time of a collision. For wave-lengths shorter or longer than this characteristic wave-length the intensity of light will rapidly fall off. Thus one of the chief properties of thermal radiation would be explained. Moreover, if we assume that the time of collision at a given temperature is independent of the nature of the radiating body, and is only depending on the absolute temperature in such a way as to decrease with increasing temperature, we understand why the color of the light for which the intensity of radiation is a maximum depends only on the temperature and moves towards the blue end of the spectrum as the temperature is increased. On this ground J. J. Thomson developed the mathematical theory of radiation and found for the energy having frequency between g and $g + dg$ radiated from unit volume in unit time the expression:

$$N\frac{8}{3\pi}\frac{e^2}{V}F_1^2 \sin^2\frac{gt_2}{2}\,dg, \tag{1}$$

$$F_1 = \int_{-\frac{t_1}{2}}^{+\frac{t_1}{2}} f(t)\cos gt\,dt. \tag{2}$$

t_1 = time of collision.

t_2 = time interval between two collisions.

N = Number of collisions in unit volume and unit time.

e = elementary charge.

V = velocity of light.

$f(t)$ = acceleration of the electron in the neighborhood of an atom as a function of time.

Considering the steady state of radiation where the amount of energy radiated from unit volume in unit time is equal to the energy

absorbed and introducing for the conductivity σ of the metal for short waves the value:

$$\sigma = 2\frac{Ne^2}{m}\frac{\sin^2\left(\frac{gt_2}{2}\right)}{g^2}.$$

J. J. Thomson finally finds the expression

$$E_g = \frac{1}{3}\frac{mk}{\pi^2 V^3} F_1^2 g^2 dg.$$

m = mass of electron.

k = specific inductive capacity of the substance.

The final result will now depend upon the function $f(t)$.

Two functions

$$f(t) = Ae^{-\frac{t^2}{a^2}}$$

and

$$f(t) = \frac{A}{a^2 + t^2},$$

were introduced in equation (2). The first expression does not lead to any of the various forms that have been proposed as laws of distribution of the energy in the spectrum of the black body. The second function however leads to the following expression:

$$E_\lambda = \frac{16\pi}{3}\frac{aT\lambda}{\lambda^5} e^{-\frac{\pi V_a}{\lambda}}$$

of the type suggested by Lord Rayleigh. If the second law of thermodynamics holds for radiant energy, E_λ must be of the form:

$$\frac{1}{\lambda^5}\varphi(\lambda T)$$

where $\varphi(\lambda T)$ is a function of (λT) and the same for all bodies. Comparing the two last functions we see that a, the time of a collision, must at the same temperature be the same for all bodies and must be inversely proportional to the temperature, *i. e.*, to the square of the average velocity of the electrons of all bodies. Upon this assumption $E\lambda$ will be found to be of the form

$$E_\lambda = C\frac{\lambda T}{\lambda^5} e^{-\frac{c}{\lambda T}}.$$

Now this law again holds only for large values of the wave-length λ, when it reduces to the expression:

$$E_\lambda = C\frac{T}{\lambda^4}$$

while it does not agree with the experiments of Lummer and Pringsheim, Rubens and Kurlbaum and Paschen for long waves.[1] Thus the theory of Lorentz-Thomson as far as it has been developed accounts only for the radiation of long waves of heat. As Thomson's expression (1), however, admits of a large variety of functions $f(t)$, I am going to determine $f(t)$ so that Wien's expression

$$E_\lambda = C\frac{e^{-\frac{c}{\lambda T}}}{\lambda^5}$$

should result, which is the law of the distribution of energy for short waves, or of visible and ultraviolet light.

The energy having frequency between g and $g + dg$ radiated from unit volume in unit time is equal to:

$$dEg = N\frac{8}{3}\frac{e^2}{\pi V}\sin^2\left(\frac{gt_2}{2}\right)F_1^2 dg. \tag{3}$$

I assume the following form of the function F_1:

$$F_1 = \sqrt{A}\,g^{\frac{1}{2}}e^{-\frac{g\alpha}{2}}$$

$$dEg = N\frac{8}{3}\frac{e^2}{\pi V}Age^{-g\alpha}\sin^2\left(\frac{gt_2}{2}\right)dg.$$

In the steady state of radiation the energy given out from unit volume in unit time must be equal to the energy absorbed. This absorption is due to the same cause as the production of heat in the conductor when an electric current passes through it, since the waves are made up of electric and magnetic forces. When the current in a conductor is i and the electric force is equal to X, the rate at which energy is absorbed in unit volume is equal to $Xi = X^2\sigma$, σ being the specific conductivity. Now the energy E per unit volume of the conductor is equal to

[1] Rubens and Kurlbaum, Drude's Annalen (4), p. 649, 1901. Paschen, Drude's Annalen (4), p. 210, 1901.

$$\frac{X^2}{4\pi V^2} \text{ or } X^2 = 4\pi V^2 E.$$

The rate at which energy of the frequency g is absorbed is equal to

$$\sigma X^2 = \sigma 4\pi V^2 dEg.$$

In the steady state the heat radiated must be equal to the heat absorbed.

$$\sigma 4\pi V^2 dEg = N \frac{8}{3} \frac{e^2}{\pi V} Age^{-ga} \sin^2\left(\frac{gt_2}{2}\right) dg,$$

$$dEg = N \frac{2}{3} \frac{e^2 A}{\pi^2 V^3 \sigma} ge^{-ga} \sin^2\left(\frac{gt_2}{2}\right) dg.$$

When the heat produced by the current represents the work done by the electric field on the electrons while moving through their free path, J. J. Thomson deduced for σ the following value:

$$\sigma = 2N \frac{e^2}{g^2 m} \sin^2\left(\frac{gt_2}{2}\right).$$

Substituting this value in the last equation we shall have:

$$dE_g = \frac{N 2e^2 m g^3 e^{-ga} A dg}{3\pi^2 V^3 2Ne^2} = \frac{mAg^3 e^{-ga}}{3\pi^2 V^3} dg$$

$$g = \frac{2\pi V}{\lambda}$$

$$dg = \frac{2\pi V d\lambda}{\lambda^2} \text{ neglecting the sign,}$$

$$dE_\lambda = VmA\tfrac{16}{3}\pi^2 \frac{e^{-\frac{2\pi Va}{\lambda}}}{\lambda^5} d\lambda \tag{4}$$

or

$$E_\lambda = C \frac{e^{-\frac{c}{T\lambda}}}{\lambda^5}.$$

Thus we find Wien's expression on the ground of the electron theory. We have of course in agreement with the second law of thermodynamics assumed that a, a quantity which as we shall see measures the time of collision between an electron and an atom, is inversely proportional to the absolute temperature T. The two

constants C and c in Wien's law may be represented by different factors as shown by M. Planck,

$$(5) \qquad dE_\lambda = V 8\pi h \frac{1}{\lambda^5} e^{-\frac{Vh}{k\lambda T}} d\lambda.$$

Comparing the equations 4 and 5 we find:

1) $$a = \frac{h}{kT2\pi}$$

2) $$mA\frac{16}{3}\pi^2 = 8\pi h,$$

$$h = \frac{2}{3}\pi m A.$$

The most characteristic feature of Planck's theory is the constant h. The electromagnetic energy is supposed to be divided up in distinct units, each unit of electromagnetic energy being equal to hg. From the last equation we see that h is proportional to the mass m of the electron.

$$h = 6.55\ 10^{-27}\ \text{erg sec}.$$

$$m = 8.4\ 10^{-28}\ \text{gr}.$$

$$A = \frac{h}{\frac{2}{3}m\pi} = 3.64\frac{\text{cm}^2}{\text{sec}}.$$

We have seen that by substituting the function

$$F_1 = \sqrt{A}g^{\frac{1}{2}}e^{-\frac{gn}{2}}$$

into the general integral (3) we get the law of distribution of the energy for visible light. What is the acceleration of the electron that satisfies the last equation? The answer to this question is given by Fourier's theorem and the second Eulerian integral.

$$F_1 = \int_{-\frac{t_1}{2}}^{+\frac{t_1}{2}} f(t)\cos gt dt = \sqrt{A}g^{\frac{1}{2}}e^{-\frac{gn}{2}},$$

t_1 is the time of disturbance of the mean free path of the electron because of its collision with an atom. This time is theoretically infinitely longer than a, so that we may write without appreciable error

$$F_1 = \int_{+\infty}^{-\infty} f(t) \cos(gt) dt = \sqrt{A} g^{\frac{1}{2}} e^{-\frac{ga}{2}}$$

Fourier's theorem gives :

$$f(t) = \frac{1}{\pi} \int_0^{\infty} \cos(gt) dg \int_{-\infty}^{+\infty} f(t) \cos gt dt,$$

$$f(t) = \frac{\sqrt{A}}{\pi} \int_0^{\infty} \cos(gt) g^{\frac{1}{2}} e^{-\frac{ga}{2}} dg.$$

We can solve this integral by adding the imaginary function :

$$jg(t) = \frac{\sqrt{A}}{\pi} \int_0^{\infty} j \sin gt g^{\frac{1}{2}} e^{-\frac{ga}{2}} dg \; ; \; j = \sqrt{-1}.$$

$$\psi(t) = \left[f(t) + jgt \right] \frac{\pi}{\sqrt{A}} = \int_0^{\infty} g^{\frac{1}{2}} e^{-g\left(\frac{a}{2} - jt\right)} dg.$$

Now the general form of the second Eulerian integral or of the gamma function is equal to

$$\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx$$

and one of its fundamental properties is expressed by the equation :

$$\int_0^{\infty} x^{p-1} e^{-xz} dx = \frac{\Gamma_{(p)}}{z^p},$$

where z is a constant with respect to x.

Comparing our integral ψ with this equation we find

$$p - 1 = \tfrac{1}{2} \text{ or } p = \tfrac{3}{2},$$

hence

$$z = \frac{a}{2} - jt ;$$

$$\psi(t) = \frac{\Gamma_{\left(\frac{3}{2}\right)}}{\left(\frac{a}{2} - jt\right)^{\frac{3}{2}}},$$

$$\psi(t) = \frac{\Gamma_{(1,5)} \left(\frac{a}{2} + jt\right)^{\frac{3}{2}}}{\left[\left(\frac{a}{2}\right)^2 + t^2\right]^{\frac{3}{2}}}.$$

In Legendre's tables of the Γ functions I find for

$$\log \Gamma_{(1,5)} = 9{,}947545,$$

$$\Gamma_{(1,5)} = 0{,}88623.$$

From the figure (1) we read:

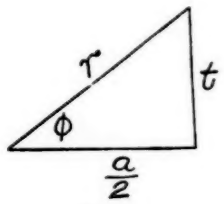

Fig. 1.

$$\frac{a}{2} + jt = r(\cos\varphi + j\sin\varphi),$$

$$r^2 = t^2 + \left(\frac{a}{2}\right)^2,$$

$$\psi(t) = \frac{\Gamma_{(1,5)} r^{\frac{3}{2}}}{r^3}\left[\cos\left(\varphi\tfrac{3}{2}\right) + j\sin\left(\tfrac{3}{2}\varphi\right)\right].$$

The two solutions are as follows:

(1) $$\psi(t) = \frac{\Gamma_{(1,5)}}{r^{\frac{3}{2}}}\left[\cos\left(\tfrac{3}{2}\varphi\right) + j\sin\tfrac{3}{2}\varphi\right].$$

(2) $$\psi(t) = \frac{\Gamma_{(1,5)}}{r^{\frac{3}{2}}}\left[\cos\left(\tfrac{3}{2}\varphi + \pi\right) + j\sin\left(\tfrac{3}{2}\varphi + \pi\right)\right].$$

It follows that:

$$f(t) = \pm\frac{\sqrt{A}}{\pi}\Gamma_{(1,5)} \cdot \frac{\cos\frac{3}{2}\varphi}{r^{\frac{3}{2}}},$$

$$g(t) = \pm\frac{\sqrt{A}}{\pi}\Gamma_{(1,5)} \cdot \frac{\sin\frac{3}{2}\varphi}{r^{\frac{3}{2}}}.$$

The solution

$$f(t) = +\frac{\sqrt{A}}{\pi}\Gamma_{(1,5)}\frac{\cos\frac{3}{2}\varphi}{r^{\frac{3}{2}}}$$

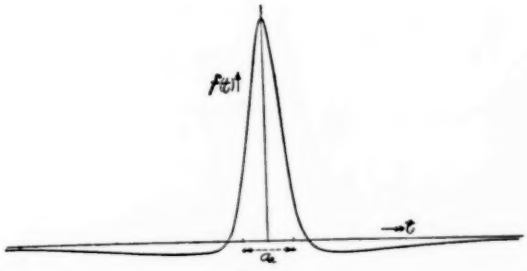

Fig. 2.

would be represented by a curve of the following shape (Fig. 2).

The t axis is an asymptote of the two branches of the curve.

The time a appears to be an approximate measure of the time of a collision, the nature of which must be of a complicated character in order to account for the curve shown in Fig. 2. At the beginning and end of a collision instead of an acceleration there appears to be an exceedingly small retardation in the motion of the electron. For $t = o$ we get :

$$\varphi = o; \quad \cos\frac{3}{2}\varphi = 1 \qquad r = \frac{a}{2},$$

$$f(t)_0 = \frac{\sqrt{A}}{\pi}\frac{I_{(1,5)}}{\left(\frac{a}{2}\right)^{\frac{3}{2}}},$$

$$\sqrt{A} = \frac{f(t)_0 \pi \left(\frac{a}{2}\right)^{\frac{3}{2}}}{I_{(1,5)}}.$$

$$A = \frac{f(t)_0^2 \pi^2 \left(\frac{a}{2}\right)^3}{[I_{(1,5)}]^2} = h\frac{3}{2\pi m}.$$

$$\text{Dimension of } f(t) = \frac{\text{cm}}{\text{sec}^2},$$

$$\text{of } a = \text{sec},$$

$$\text{of } \sqrt{A} = \frac{\text{cm}}{\text{sec}^2}\text{sec}^{\frac{3}{2}},$$

$$\text{of } A = \frac{\text{cm}^2}{\text{sec}},$$

in agreement with the dimension of A determined above.

$$h = \frac{f(t)^2 \pi^3 m a^3}{I_{(1,5)} \cdot 12}.$$

In the electron theory of radiation h does not appear to have such a simple physical significance as in the theory of Planck. It is evident however that we do not understand this quantity nor the nature of the unit of electromagnetic energy from a pure physical point of view. Planck's theory would indicate that an elementary oscillator does not receive nor emit radiant energy in a continuous

way but with interruptions, unit after unit,

$$a = \frac{h}{2\pi k T}.$$

Substituting for h and k the values given by Planck we find for a temperature $T = 1273$,

$$a = 6{,}08 \cdot 10^{-15} \text{ sec.}$$

The thickness of the Roentgen pulse corresponding to this time of collision would be equal to:

$$\delta_{1000} = V \cdot a = 18{,}24 \cdot 10^{-5} \text{ cm.}$$

For a temperature $T = 273$ or 0° C., a would be equal to $2{,}83\ 10^{-14}$ and the thickness of the corresponding Roentgen pulse would be equal to:

$$\delta_0 = Va = 8{,}49 \cdot 10^{-4} \text{ cm.}$$

As the thickness of the Roentgen pulse only depends on the time of a collision between the electron and an atom and as this time is the same for all bodies at the same temperature and is inversely proportional to the temperature we should expect that the thickness of Roentgen rays due to the impact of electrons on the anticathode in the Roentgen tube is independent of the substance and temperature of the metal struck by the cathode rays. The actual phenomena however will be complicated by secondary cathode rays, starting from the anticathode.

THE ELECTRICAL AND OPTICAL PROPERTIES OF METALLIC SELENIUM.

BY A. H. PFUND.

IN an earlier paper[1] I published the results of some experiments which were carried out to determine the variations in resistance of metallic selenium[2] with the wave-length of the incident light. It was found that while the blue and infra-red regions of the spectrum had but little effect, light of wave-length $\lambda = 700\ \mu\mu$ produced a very large change in resistance. In fact the so-called "sensibility curve" showed an extremely pronounced maximum at this point. It was then suggested that some relation might exist between the electrical and optical behavior of selenium, but, inasmuch as no data on the optical properties were available, it was impossible to draw any conclusions. In this paper the absorption and reflection curves of selenium are given and a more exact determination of the sensibility curve is presented. While the reflection and absorption curves show no maxima in the region of the maximum sensibility, it nevertheless seems possible to account for the existence of this maximum by taking into consideration the peculiarities presented by the electrical conduction in thin films. A tentative explanation is offered to account for the phenomena observed and experiments are presented in support of the explanation.

Unfortunately other work has made it necessary that the investigation of selenium be temporarily discontinued, and under these conditions it seemed advisable to publish the results thus far obtained. At the very earliest opportunity, I hope to return to the work.

SELENIUM CELLS.

The process employed in the making of selenium cells is, in reality, very simple. A piece of plane glass $1 \times 3 \times 0.1$ cm. is ground

[1] Phil. Mag., 7, p. 26, 1904.

[2] Unless otherwise specifically mentioned, it is understood that the metallic modification is being discussed whenever the term "selenium" is used.

with emery on one side and some highly purified amorphous selenium is spread over the ground surface by means of a hot glass rod. The thickness of layer is such that in transmitted light the film, in its thinner portions, appears deep ruby red. Next, four strands of no. 30 bare copper wire are wound around the cell in such a manner that the entire selenium surface is covered. After binding the loose ends of the wires to the glass plate, the second and fourth wires are unwound, thus leaving the first and the third separated by a distance equal to the diameter of a wire. To make the cell light-sensitive, it is placed in an air-bath, previously heated to 180° C. and is permitted to remain there until the transformation from amorphous to metallic selenium has taken place — an interval of time not exceeding five minutes. Where this transformation is complete, the cell is taken out and is permitted to cool down to room temperatures. Although this procedure this probably not the one yielding cells of the highest sensibility, it nevertheless has proved entirely trustworthy thus far, as every cell has been a success. The resistance of these cells is of the order of 2×10^7 ohms and the sensibility is such that a 16 c.p. incandescent lamp, at a distance of 30 cm., increases the conductivity of the cell by about ten times. In order to protect the cells from moisture (thus making them permanent), they are either placed in a glass tube which is subsequently exhausted on a Geryk oil pump or are fastened to a piece of plane glass or mica with soft wax.

Sensibility Curve.

In order to determine the variations of electrical resistance of selenium with wave-length of incident energy, the method of procedure followed out was practically the same as that employed in my previous work.[1] By means of an ocular slit, a narrow region of the spectrum was isolated and permitted to fall on a selenium cell, whose change in resistance was recorded by galvanometer deflections. As before, the energy carried by each bundle of approximately homogeneous radiations before being permitted to fall on the selenium cell, was reduced to the same value in each case — thus making the results obtained independent of the distribution of

[1] Pfund, l. c.

energy in the spectrum of the source employed. Instead of recording, as previously, the throw at the galvanometer spot of light after an exposure of the selenium cell to light for ten seconds, in the present work the maximum deflection was taken, even though the time of exposure had to be prolonged to two or three minutes or more. The arrangement of apparatus is shown in Fig. 1.

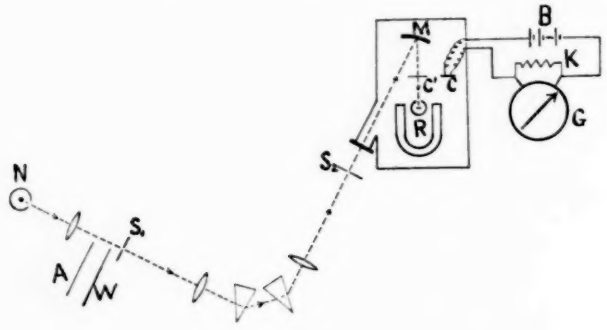

Fig. 1.

The light from a Nernst lamp at N, after being concentrated on the slit S_1 of the spectroscope, was resolved into a spectrum, of which a narrow portion was transmitted by the slit S_2. This isolated bundle of homogeneous rays was reflected from a short-focus concave mirror at M and was finally concentrated on the blackened junction of a Boys radiomicrometer R. At will, the selenium cell c could be moved to the position c' so as to intercept the beam of light. In order to vary the intensity of the light, a smoked glass "optical wedge" was introduced at W, and a screen for cutting off the light entirely, at A. The selenium cell was connected in series with a 20-volt storage battery B and a dead-beat D'Arsonval galvanometer G. A shunt at K made it possible to vary the galvanometer sensibility from 3×10^{-9} amperes, down. As previously stated, the energy of each bundle of homogeneous radiations, before being permitted to fall on the cell, was, in each case, reduced to such an amount that it produced a radiomicrometer deflection of 30 mm. The selenium cell was then exposed until a maximum galvanometer deflection was reached. The results obtained are given in the table below (Table I.) and are plotted in Fig. 2.

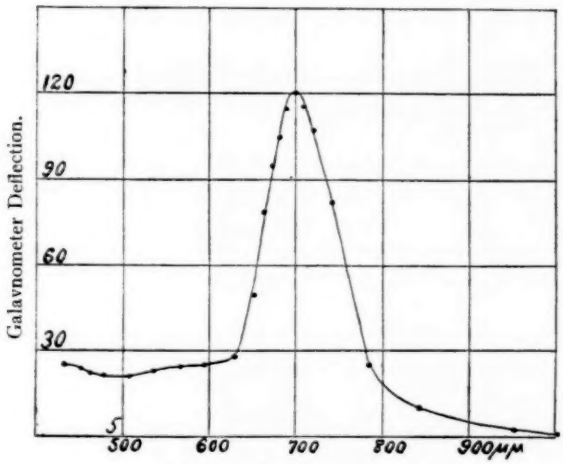

Fig. 2. Wave-length.

TABLE I.

λ in μμ.	Galv. Defl.	λ in μμ.	Galv. Defl.
435	25 mm.	683	106 mm.
454	24	692	115
463	22	700	120
482	21	711	116
511	21	722	108
535	23	745	83
570	24	787	25
597	25	842	9
630	28	950	2
655	50	1005	1
667	80		
675	96		

REFLECTION CURVE.

The mirror of metallic selenium was made by pressing a hot glass plate into contact with a pool of molten selenium which was then transformed into the metallic state. After many unsuccessful attempts it was found that the selenium layer had to be at least 2 mm. thick in order to avoid the deformations of the reflecting surface which otherwise appear when a thinner layer is transformed into the metallic state. If the process of transformation has been successful, it is found that the selenium mirror has let go entirely of the glass and

has an excellent, "plum-colored" optical surface. The mirror actually used had an area 8 × 12 mm. — and an optical surface which left little to be desired.

Measurements of the reflecting power were obtained by determining the ratio of the radiomicrometer deflections when light was reflected from the selenium mirror, and then, from a silver mirror — the angle of incidence in both cases being 5°. Knowing the reflecting power of silver,[1] the true reflecting power of selenium is obtained at once. The method of mounting the mirrors and the general details of the procedure were identical with those described in an earlier paper[2] and, therefore, need not be repeated, here. The results obtained are given in Table II., and are plotted in Fig. 3.

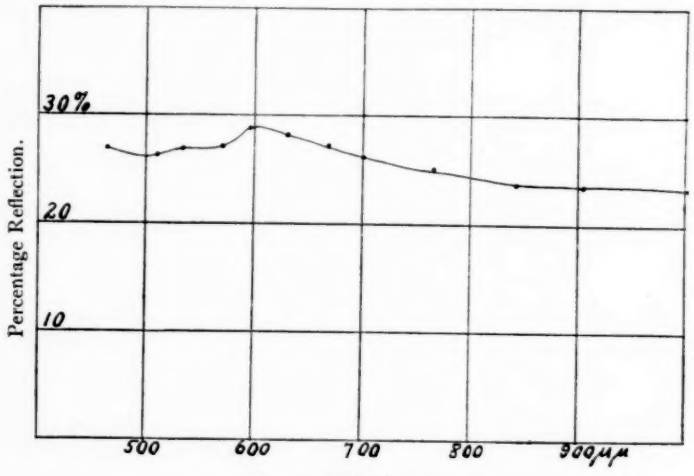

Fig. 3. Wave-length.

ABSORPTION CURVE.

Films of amorphous selenium, which appeared red by transmitted light, became absolutely opaque upon being transformed into the metallic state. The only procedure which seemed available for making thin films of metallic selenium was the one by which the selenium is deposited by cathode discharge. In order to produce such films the usual type of vacuum chamber was used — the glass plate upon which the selenium was to be deposited being placed

[1] Hagen and Rubens, Annalen d. Physik, 8, p. 1, 1902.
[2] Pfund, Ap. Jour., XXIV., 1, p. 19, 1906.

TABLE II.

λ in μμ.	Deflections in mm.		Percentage Reflecting Power.	
	Sel.	Ag.	Uncorrected.	Corrected.
463	28.2	95.0	29.7	27.0
511	35.7	125.0	28.6	26.3
535	45.7	156.0	29.3	27.0
570	34.3	115.7	29.6	27.2
597	38.8	125.0	31.0	28.7
630	34.8	115.5	30.2	28.1
667	35.7	123.0	29.1	27.3
700	38.2	137.0	27.8	26.1
765	37.7	141.5	26.7	25.2
842	39.1	155.2	25.3	23.8
905	37.0	149.0	24.8	23.7
1005	38.0	155.0	24.5	23.5

2.5 cm. below the upper (aluminium) electrode which had previously been covered with metallic selenium. Heretofore it has been customary to employ the current from the secondary of an induction coil, or from a high potential storage battery. It seemed to me that, since the base of the vacuum chamber was an aluminium plate (which served as the other electrode) and since aluminium is not deposited, it might be possible to use a high potential alternating current to produce the deposit of selenium. Consequently, the current supplied by the secondary of a small transformer was used. By connecting the secondary which gave 1,000 volts, to the discharge vessel it was found that a heavy deposit of selenium was produced in two minutes. Unfortunately, the film was granular and hence unsuitable; however, hard, continuous films were obtained by introducing resistance into the primary circuit and thus cutting down the secondary current until the time of deposit was increased to about four hours. Curiously enough, while the selenium on the electrode was in the metallic state, the selenium deposited on the glass plate was in the amorphous state.

A great source of difficulty was encountered when it was attempted to transform the film into the metallic state. In spite of every precaution, the selenium, upon being heated, would persist in drawing itself up into small globular masses — thus making the film discontinuous. Although the proper conditions for producing

metallic films have not as yet been found, I did succeed, by pure chance, in obtaining a film which was in every way satisfactory. This film, which had the dimensions $.9 \times 1.2 \times 4.5 \cdot 10^{-6}$ cm. appeared grayish brown in transmitted, and plum-colored in reflected light.

Ordinarily, when absorption measurements are carried out on thin films, it is customary to employ two films of different thickness so as to eliminate all other effects except those occasioned by the change in absorption of a film having a thickness equal to the difference in thickness of the two films in question. Inasmuch as but one film was available, it was necessary to make observations on, and later, corrections for the various losses by reflection. (The losses due to absorption in the glass plate were found to be negli-

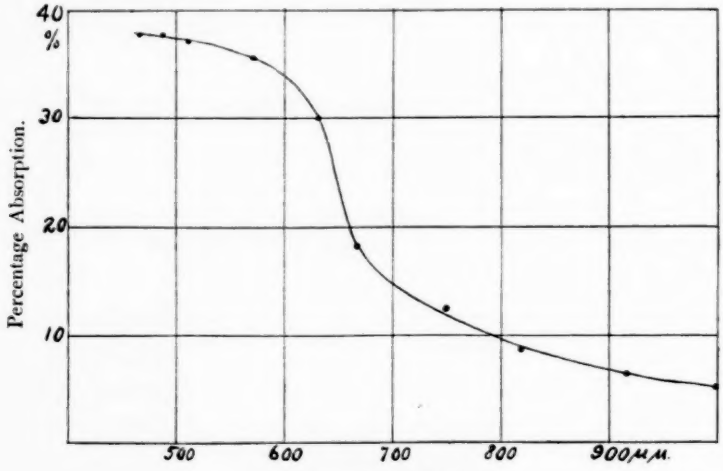

Fig. 4. Wave-length.

gible.) Furthermore, in the determination of film thickness by means of interference fringes, the effects of phase change are similarly eliminated by employing two films of different thickness. Since but one film was at hand, the measurements here given are, necessarily, not as exact as those which could be obtained from two films. As soon as I shall have succeeded in obtaining the desired films, I hope to repeat this work. For the present, however, the results are sufficiently accurate to make it permissible to draw conclusions from them.

The observations on absorbing power were obtained in the usual manner by the use of the spectrometer and radiomicrometer. As previously stated, the film thickness was found equal to one tenth of the wave-length of blue light, *i. e.*, 4.5×10^{-6} cm. The results are plotted in Fig. 4, where the terms "Percentage Absorption" are defined by the expression

$$1 - \frac{T}{I(1 - R)},$$

where I is the intensity of the incident light, T the intensity of the transmitted light, and R the percentage reflection of the system.

Discussion of Results.

For the ease of discussion, the reflection, absorbtion, and sensibility curves are plotted together in Fig. 5. From these curves it is evi-

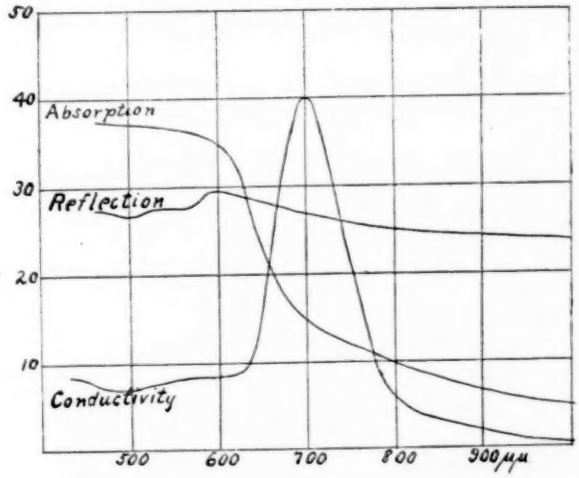

Fig. 5.

dent that no marked maxima or minima of absorption or reflection occur near 700 $\mu\mu$. As the optical properties by themselves fail to account for the sensibility maximum, I have been led to take into account another effect which thus far, I think, has not been connected with this problem.

At the present time it is quite generally acknowledged that the conduction in selenium is electronic and not electrolytic in character.

Selenium cells which I had made several years ago (and which had been preserved in an evacuated bulb) failed to show the least traces of polarization, after having been connected for several hours with a 110-volt direct current dynamo. Again, cells were made from highly purified selenium, which, in one instance, was deposited on glass by distillation in vacuo, and in the other by cathode discharge. Both cells conducted and showed sensibility. Although by no means definitely proved, it seems very probable that the changes in resistance upon exposure to light are due to the selenium itself and not to selenides, which are present as impurities. This view is supported by my earlier experiments, which showed that the position of maximum sensibility is uninfluenced by the chemical nature of the selenide which had been mixed with the selenium. For the present, then, it will be considered that the conduction is electronic, — the selenium itself supplying the electrons.

Upon this view it is considered that light, in falling upon selenium and in being absorbed, sets up resonance within the atom, and this, in turn, gives rise to an expulsion of electrons, — thus increasing the number of electrons available for carrying the current, and hence, increasing the conductivity.[1] Inasmuch as absorption and resonance go together, it might be expected, in passing through the spectrum from the infra-red toward the shorter wave-lengths, that, as the absorption continuously increases, the conductivity would also continuously increase, yielding the largest values in the blue where the absorption is the greatest.[2] This deduction is not verified by experiment, and it is at this point that the new element, previously alluded to, enters into the problem.

Longden,[3] Vincent[4] and Patterson[5] have shown that the specific resistance of metallic films remains constant until a thickness of about 5×10^{-6} cm. is reached, after which the specific resistance increases very rapidly with a decrease in thickness. J. J.

[1] It is held the change in conductivity is the internal manifestation on an effect whose external manifestation is known as the "photo-electric effect."

[2] Since the reflecting power does not suffer marked changes, it is not necessary to take into consideration the variations in the amounts of energy which actually penetrate into the selenium.

[3] PHYS. REV., Vol. XI., p. 40, 1900.

[4] Annales de Ch. et Physique, 7, XIX., p. 421, 1900.

[5] Phil. Mag., 56, p. 652, 1902.

Thomson then showed that this rapid increase in resistance could be accounted for on the assumption that the change sets in when the thickness of film becomes less than the mean free path of the conducting electrons within the metal. The curve for silver, as obtained by Patterson, is given below in Fig. 6. Attention is to be called to the fact that the results of the various observers are not entirely in accord as to the "critical" thickness at which the change of resistance sets in. Again, the results are not the same for different metals or even for the same metal (at least as inferred from the variations in specific resistance) when the films are produced by different methods. All of these points taken into consideration make it safe, I think, to place the critical thickness at a value which is of the order of magnitude of 10^{-6} cm.

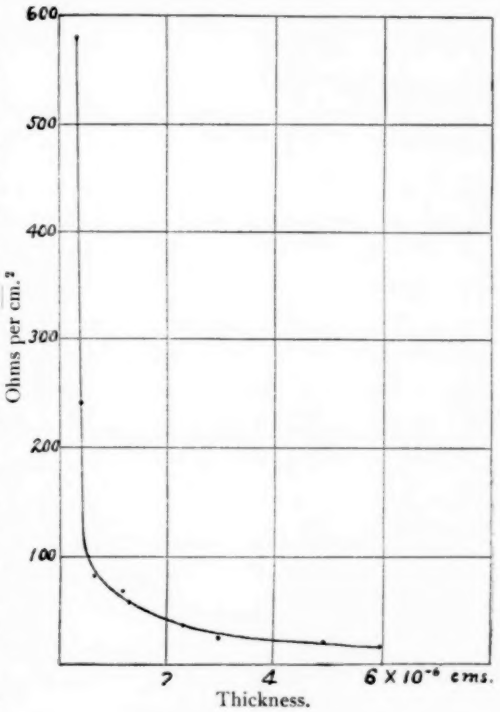

Fig. 6.

These results appear to apply directly to the conduction in a selenium cell. The absorption curve shows that the incident energy is largely absorbed in a film whose thickness is of the order of magnitude of the "critical" thickness. Furthermore, since the conductivity of a cell may easily be increased ten times or more by illumination and since the thickness of surface film carrying this increased current is infinitesimal in comparison to the thickness of the selenium layer (which in this case was about 0.01 mm.), it is evident that the conductivity of the surface film must be comparatively high. This state of affairs is, then, considered comparable

with that of a thin metallic film deposited on glass. Now then, beginning in the infra-red, where the depth of penetration of radiation is greater than the "critical" thickness, it is observed (Fig. 5) that the conductivity increases rapidly with increased absorption. However, in passing through the region of $\lambda = 700\,\mu\mu$ where the absorption increases very rapidly, the depth of penetration, and hence, the thickness of current-carrying layer becomes equal to, and then, less than the critical thickness. It is in this region that the enormous decrease in conductivity with thickness makes itself felt and not only neutralizes the effects due to absorption, but actually causes a sharp drop in conductivity.

In using the terms "depth of penetration" it is tacitly assumed that the light is effective in producing changes in resistance only until the amplitude has died down to a certain minimum value, below which its effectiveness is lost. This fits in with the resonance theory, for the increased amplitude due to resonance must exceed a definite minimum value before electrons are expelled from the atom.

It follows as a necessary consequence of this explanation, that, if the wave-length at which the maximum of sensibility occurs, depends upon the depth of penetration of light, then, as the intensity of the light (and hence, its penetration) is changed, the position of the maximum ought to shift — the shift being toward the shorter wave-lengths with increasing intensity. The experiment devised to test this consequence was carried out as follows: A single reflecting surface of plane glass, mounted behind the ocular slit of the spectrometer projected upon the selenium cell a bundle of monochromatic radiations whose intensity was but 5 per cent. of that incident upon the glass reflecting surface. (Any scheme involving the use of a rotating sector, or of varying the position of the cell in the diverging beam of light would have been objectionable for obvious reasons). Without reducing the intensities of the various beams of monochromatic light to the same values, as before, galvanometer deflections, proportional to changes in resistance, for various wave-lengths in the region of maximum sensibility, were obtained. Next, the glass mirror was removed and the selenium cell was placed in the direct path of the beam emergent from the spectrometer and a series of readings similar to the preceding ones was obtained. Care

was taken to see that such elements as the current passing through the cell, order of magnitude of galvanometer deflections, etc., were the same in both cases — thus leaving the single point of difference that in one case the intensity of the energy falling on the cell was but one twentieth of the other. The results obtained are plotted in Fig. 7 where the solid curve (*a*) corresponds to the changes in re-

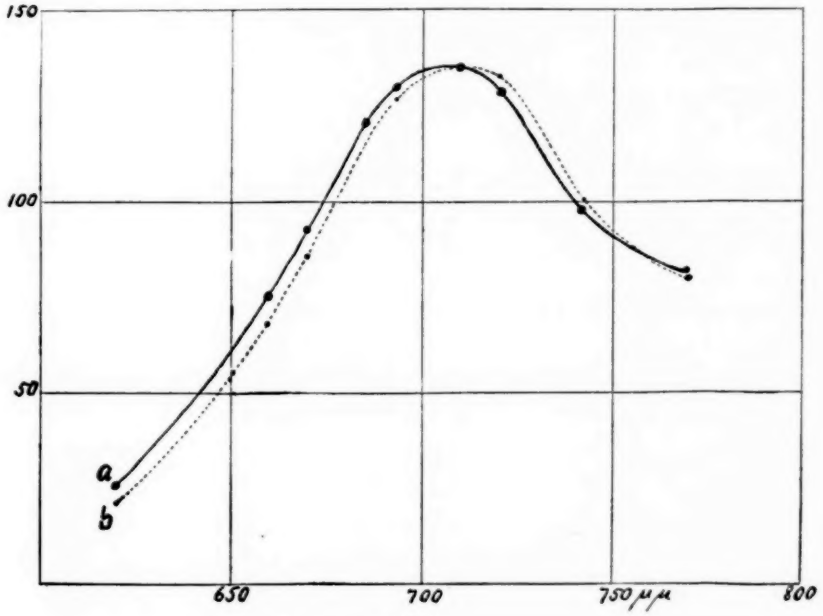

Fig. 7.

sistance for the intense light, and the dotted curve (*b*), for the weak light. It is at once apparent that by increasing the intensity of light, the position of the maximum in curve (*a*) has been shifted toward shorter wave-lengths. This observation is entirely in accord with previous deductions.

In carrying out these measurements it was noted that a marked difference existed in the time required by the resistance of the cell to reach an equilibrium condition when the intensity of light was varied. The time required by the intense light was certainly not greater than 15 seconds (time required by galvanometer to reach steady deflection), while, for the weak light, the time had to be prolonged to about 3 minutes. Again, the necessary time of exposure

was less for the wave-lengths lying on the blue side of the maximum, than for those lying on the red side. These observations seem suggestive when it is recalled that Lenard[1] and others[2] found similar results in their study of the photo-electric effect when intense and weak sources of light were used. It is my intention to study, in the near future, the photo-electric behavior of selenium for different wave-lengths. On the basis of the theory proposed to account for the behavior of the selenium cell one might expect that the maximum of photo-electric action would not lie in the red but rather in the violet, where the absorption is great.

There is yet another experiment to test the correctness of the previously mentioned explanation. If it were possible to make a cell so thin as to be transparent to light, then the thickness of layer would be less than the "critical" thickness — and hence the maximum of sensibility in the red might be expected to be absent. An attempt was made to transform the transparent film, used in the absorption measurements, into a cell, but, due to poor electrical contacts and the high resistance of the film, not enough current could be forced through the system to make the experiment successful. It was felt that some time would elapse before satisfactory thin-film cells could be made, hence it was decided, in view of other pressing work, to discontinue the investigation for the time being.

SUMMARY.

The results presented may be briefly summarized as follows:

1. The conductivity or "sensibility" curve of metallic selenium has been redetermined.

2. The absorption and reflection curves have been determined in the interval extending from about 450 $\mu\mu$ to 1,000 $\mu\mu$.

3. An explanation, involving the peculiarities presented by conduction in thin films, has been offered to account for the existence of the maximum of sensibility.

4. It was found that the position of maximum sensibility shifts toward the shorter wave-lengths as the intensity of light is increased. This experiment is exactly in accord with the theory proposed.

JOHNS HOPKINS UNIVERSITY,
March, 1909.

[1] Lenard, Ann. d. Phys., II., p. 355, 1900.

[2] Millikan & Winchester, Phil. Mag., 14, p. 188, 1907.

THE NUMBER OF FREE CORPUSCLES PER UNIT VOLUME OF THE METALS GOLD, PLATINUM AND SILVER.

By B. J. Spence.

SEVERAL writers have attempted to establish the upper and lower limits to the number of free corpuscles per unit volume of the metals, but conflicting results appear, depending upon the manner in which the values have been obtained.

In his recent book on "The Corpuscular Theory of Matter" J. J. Thomson describes several methods by which the limits to the number of free corpuscles have been obtained. One method described by him has led to negative results owing to experimental difficulties. Another method has led to a lower limit. It is based upon the work of Rubens[1] and Hagen dealing with the reflection of the longer heat rays from the surface of the metals. These writers show that the conductivity of the various metals for periodic disturbances corresponding to a wave length $\lambda = 250 \times 10^{-5}$ cm. is practically that observed in the case of steady currents. These results in conjunction with an expression deduced by Thomson, has led Thomson to assign a value of 1.8×10^{24} as the number of free corpuscles per unit volume for silver.

This result however leads to a difficulty. By a consideration of the kinetic theory of gases Thomson shows that if this result be correct, then the energy necessary to raise the temperature of one cubic centimeter would be greater than 2.7×10^{8} ergs or about 6 gram calories. But it is known that the energy necessary to raise the temperature 1° C. of atoms as well as corpuscles is only .6 gram calorie.

Another method involving the time of duration of free path of the corpuscle is due to Bloch,[2] who has taken the above-mentioned expression of Thomson, and so combined it with the Maxwell equa-

[1] Phil. Mag., 7, 157, 1904.
[2] Comptes Rendus, CXLV., 1907, p. 754.

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tion $n^2\varkappa = \sigma_\infty \tau$, where n is the refractive index of the metal, $\varkappa$ its extinction coefficient, σ_∞ the conductivity for steady currents, and τ the period, that he is able to obtain a value of the number of free corpuscles per unit volume, and also a value of the time of duration of free path. The values of N, at which he arrives, are of the order of 10^{24} per cubic centimeter for a number of the metals, and the values of the time of duration of free path are of the order of 10^{-15} sec.

These results do not agree with those obtained by Schuster,[1] who found that the number of corpuscles must be of the order of magnitudes of the number of atoms per unit volume, which is about 10^{23}. Schuster's analysis is based upon the optical properties of the metals and does not involve the time of duration of free path explicitly. His derivation leads simply to the ratio of the number of atoms to the number of corpuscles per unit volume, and is based upon underlying principles similar to those of Drude,[2] upon which the method of this paper is based. We will give Drude's analysis somewhat in detail.

Drude assumed that in the metals there are two kinds of corpuscles, the vibrating and the conducting corpuscles. The equation of motion for the vibrating or bound corpuscle is given by

$$m_1 \frac{d^2x_1}{dt^2} = e_1 X - \frac{4\pi e_1^2}{\theta_1} X_1 - r_1 e^2 \frac{dx_1}{dt}. \qquad (1)$$

The first term on the right-hand side represents the total impressed force, the second term is the elastic force called into play to bring the corpuscle back to its original position, and the third term is a frictional force which opposes the motion of the corpuscle.

The equation of motion of the conducting corpuscle is like the equation of the vibrating corpuscle, except that for the conducting corpuscle the term containing the elastic force vanishes, and the equation then is

$$m \frac{d^2x}{dt^2} = eX - re^2 \frac{dx}{dt}. \qquad (2)$$

Drude next finds an expression for the current density due to the presence of both kinds of corpuscles. It consists of four terms, the

[1] Phil. Mag., 7, 151, 1904.
[2] Drude's Ann., 14, 936, 1904.

first representing the displacement current in the æther, the second and third representing the current produced by positive and negative corpuscles respectively, and the fourth, the current due to the conducting corpuscles.

The complete expression for the x component of the total current density is then

$$J_x = \frac{1}{4\pi}\frac{dx}{dt} + \frac{d}{dt}(e_1 N_1 x_1 + e_2 N_2 x_2) + \mu_e. \tag{3}$$

By the use of equations (1), (2) and (3), and the fundamental conditions implied by the Maxwell equations, Drude arrives at a complete theory of dispersion, based solely upon the above underlying assumptions.

Without going into further detail we may write the final equation obtained by Drude, for the current density along the x axis, which he obtained by regarding the impressed force, and likewise the displacement x, as periodic.

$$J_x = \frac{1}{4\pi}\frac{dX}{dt}\left[1 + \Sigma\frac{N_\lambda\theta_\lambda}{1 + \frac{r_1\theta_1}{4\pi T} - \frac{m_1\theta_1}{4\pi e^2 T^2}} + \Sigma\frac{4\pi T N_1}{ir_\lambda - \frac{m}{e^2 T}}\right], \tag{4}$$

where $T = 2\pi/\tau$, τ being the period.

By a consideration of the Maxwell equation we may at once write the expression for the complex dielectric constant of the medium under consideration.

$$\varepsilon = 1 + \Sigma\frac{N_\lambda\theta_\lambda}{1 + \frac{r_1\theta_1}{4\pi T} - \frac{m_1\theta_1}{4\pi e^2 T^2}} + \Sigma\frac{4\pi T N_1}{ir_\lambda - \frac{m}{e^2 T}}, \tag{5}$$

where ε may also be written in terms of refractive index and absorption coefficient, as

$$\varepsilon = n^2(1 - ia)^2.$$

If we separate real and imaginary quantities and simplify, we obtain Drude's final equations

$$na = \Sigma\frac{r_\lambda \lambda N_1}{c\left(r_\lambda^2 + \frac{4\pi m^2 c^2}{e^4\lambda^2}\right)}, \tag{6}$$

$$n^2 - a^2 + 1 = \Sigma \frac{\theta_\lambda N_\lambda \lambda^2}{\lambda^2 - \lambda_0^2} - 4\pi \Sigma \frac{N_1 \frac{m}{e^2}}{c\left(r_\lambda^2 + \frac{4\pi m^2 c^2}{e^4 \lambda^2}\right)}, \qquad (7)$$

where c is the velocity of light in vacuo, λ_0 the wave-length corresponding to the free period of the vibrating corpuscle, and λ the wave-length for which the values of n and a are known.

Our present knowledge of the optical properties of the metals platinum, silver and gold, lead us to the conclusion that they contain at least one kind of vibrating corpuscle, which is indicated by the consideration of the reflection coefficients obtained for the visible spectrum by Rubens[1] and Hagen. An indication of the existence of such a corpuscle comes also from the consideration of the values of the refractive indices of these metals obtained by the writer.[2] No indication of the existence of other kinds of vibrating corpuscles has been obtained for these metals, for their reflection coefficients seem perfectly regular even to very great wave-lengths.

If then in the further application of Drude's equations we are dealing with wave-lengths which are removed from the wave-length which corresponds to the vibrating corpuscle, the contribution to equation (7), due to the influence of the vibrating corpuscle will be small, compared with the second term, which is the contribution due to the conducting corpuscle. We may thus neglect the term expressing the contribution due to the vibrating corpuscle, and we then obtain two equations containing two unknown quantities, N and r_λ. The equations are

$$na = \frac{r_\lambda \lambda N_1}{c\left(r_\lambda^2 + \frac{4\pi m^2 c^2}{e^4 \lambda^2}\right)}, \qquad (8)$$

$$a^2 - n^2 + 1 = \frac{4\pi N_1 \frac{m}{e^2}}{c\left(r_\lambda^2 + \frac{4\pi m^2 c^2}{e^4 \lambda^2}\right)}. \qquad (9)$$

The effect of neglecting the term expressing the influence of the vibrating corpuscle will be noticed in the applications of these

[1] Phil. Mag., 7, 157, 1904.
[2] PHYS. REV., 28, April, 1909.

equations as we near the absorption bands in the visible spectrum, but since the results which follow begin at a wave-length 6×10^{-5} cm. the effect of dropping the term must necessarily be small.

It now remains to solve equations (8) and (9) for r_λ, where r_λ is defined by $\sigma_\lambda = N/r_\lambda$, in which σ_λ is the conductivity of the metals for a frequency corresponding to the wave-length λ, and also to solve for the quantity N_1, which expresses the number of free corpuscles per unit volume.

If we divide equation (8) by equation (9), and simplify we obtain a value of r_λ, which is given by the following equation

$$r_\lambda = \frac{4\pi mc}{e^2} \frac{na}{\lambda(a^2 - n^2 + 1)} = 2.3 \times 10^3 f(\lambda). \qquad (10)$$

If now we substitute this value of r_λ in equation (8), we obtain an expression for N which is

$$N_1 = (a^2 - n^2 + 1) \frac{mc^2}{e^2} \left(4\pi[f(\lambda)]^2 + \frac{1}{\lambda^2}\right),$$

$$= .55 \times 10^{13}(a^2 - n^2 + 1) \left(4\pi[f(\lambda)]^2 + \frac{1}{\lambda^2}\right). \qquad (11)$$

The numerical values of the constant multipliers in (10) and (11) were obtained by substituting the known values of $e = 3 \times 10^{-10}$ C.G.S. electrostatic units, $m = .55 \times 10^{-27}$ gm., and $c = 3 \times 10$ cm./sec. We thus arrive at two equations, one determining r_λ as a function of the wave-length, and the other giving the values for the number of free corpuscles per unit volume in the metals.

Results.

If in equation (10) we substitute values for a and n, we should obtain a value of r_λ for any given wave-length for which a and n are known. From the work of Rubens[1] and Hagen we may obtain the values of a, the absorption coefficient, and from the work of the writer,[1] we may obtain the values of the refractive indices for the metals, gold, platinum and silver to a wave-length 17×10^{-5} cm.

The values of r_λ are given in Tables I., II. and III. for these metals, and likewise the values of $f(\lambda)$. Having thus found the

[1] Loc. cit.

TABLE I.

Silver.

1 Wave-length in cm.	2 n	3 n^2	4 a^2	5 a^2-n^2+1	6 na	7 $\frac{na}{a^2-n^2+1}$	8 $f(\lambda)$	9 $r_\lambda=Pf(\lambda)$	10 N	11 $\sigma_\lambda=\frac{N}{r_\lambda}$ $N=.37\times10^{23}$
6×10^{-5}	.36	.111	17.64	18.53	1.51	.0815	.0136 $\times10^5$	31.3 $\times10^5$	$.293\times10^{23}$	11.8×10^{15}
7 "	.43	.185	30.47	31.29	2.37	.0757	.0108 "	24.84 "	.35 "	14.9 "
8 "	.52	.271	37.70	38.43	3.23	.0840	.0105 "	24.15 "	.33 "	15.4 "
9 "	.55	.304	52.56	53.26	3.98	.0750	.0083 "	19.09 "	.36 "	19.4 "
10 "	.61	.372	64.0	64.63	4.88	.0755	.00755 "	17.17 "	.355 "	21.3 "
11 "	.65	.422	83.0	83.58	5.94	.0710	.00645 "	14.82 "	.380 "	25.0 "
12 "	.66	.435	110.0	110.57	6.63	.0600	.00500 "	11.25 "	.41 "	32.8 "
13 "	.68	.462	118.8	117.34	7.41	.0631	.00485 "	11.08 "	.38 "	34.3 "
14 "	.70	.490	138.2	138.71	8.25	.0599	.00427 "	9.85 "	.39 "	37.6 "
15 "	.72	.518	154.2	154.68	8.91	.0576	.00384 "	8.72 "	.37 "	42.5 "
40 "						extrapolated	.00274 "	6.32 "		60.0 "
∞ "						"	.00265 "	6.10 "		61.0 "

$\sigma_\infty = 60.9\times10^{15}$ observed.

TABLE II.

Gold.

1 Wave-length in cm.	2 n	3 n^2	4 a^2	5 a^2-n^2+1	6 na	7 $\frac{na}{a^2-n^2+1}$	8 $f(\lambda)$	9 $r_\lambda = Pf(\lambda)$	10 N	11 $\sigma_\lambda = \frac{N}{r_\lambda}$ $N = .33\times 10^{23}$
5.89×10^{-5}	.45	.2025	7.95	8.75	1.27	.154	$.0261\times10^{15}$	60.03×10^{5}	$.173\times10^{23}$	5.5×10^{15}
6 "	.48	.231	8.49	9.26	1.39	.152	.0250 "	57.5 "	.181 "	5.75 "
7 "	.60	.360	17.06	17.70	2.47	.139	.019 "	43.7 "	.228 "	7.56 "
8 "	.71	.504	26.94	27.44	3.68	.134	.017 "	39.1 "	.286 "	8.45 "
9 "	.77	.592	36.00	36.41	4.62	.127	.014 "	32.2 "	.294 "	10.3 "
10 "	.82	.672	47.61	47.94	5.65	.118	.012 "	27.6 "	.311 "	11.9 "
11 "	.85	.722	60.44	61.14	6.62	.109	.0099 "	24.8 "	.333 "	13.3 "
12 "	.87	.755	78.33	78.58	7.69	.098	.0082 "	18.8 "	.336 "	17.5 "
13 "	.89	.791	94.04	94.35	8.63	.0916	.00705 "	16.2 "	.338 "	20.3 "
14 "	.90	.810	110.25	110.44	9.45	.0860	.00610 "	14.03 "	.344 "	23.5 "
15 "	.91	.828	127.69	127.87	10.03	.0772	.00517 "	11.8 "	.302 "	28.0 "
16 "	.93	.865	146.41	146.55	11.27	.0769	.00480 "	11.0 "	.338 "	30.0 "
17 "	.94	.882	169.0	169.12	12.20	.0735	.0043 "	9.89 "	.345 "	33.5 "
18 "	.95	.902	187.62	187.72	13.10	.0700	.0039 "	8.97 "	.369 "	36.8 "

$\sigma_\infty = 40\times10^{15}$.

TABLE III.

Platinum.

1 Wave-length in cm.	2 n	3 n^2	4 a^2	5 a^2-n^2+1	6 an	7 $\frac{an}{a^2-n^2+1}$	8 $f(\lambda)$	9 $r_\lambda=Pf(\lambda)$	10 N	11 $N=.81\times10^{23}$ $\sigma_\lambda=\frac{N}{r_\lambda}$
5.89×10^{-5}	1.75	3.06	16.65	14.59	7.14	.490	$.083\times10^5$	$.211\times10^5$	$.92\times10^{23}$	3.85×10^{15}
6 "	2.40	5.76	17.30	13.54	9.97	.736	.123 "	.283 "	1.62 "	2.9 "
7 "	3.45	11.9	23.13	12.23	16.57	1.355	.192 "	.4416 "	3.25 "	1.8 "
8 "	4.1	16.8	28.83	13.03	21.90	1.681	.210 "	.483 "	3.47 "	1.7 "
9 "	4.3	18.5	34.26	16.78	25.60	1.526	.189 "	.4347 "	4.25 "	1.86 "
10 "	4.4	19.36	41.86	23.50	28.41	1.633	.163 "	.3749 "	4.40 "	2.15 "
11 "	4.5	20.25	46.92	27.67	30.80	1.113	.101 "	.2323 "	2.39 "	3.5 "
12 "	4.62	21.34	61.65	41.31	36.30	.878	.073 "	.1748 "	1.67 "	4.63 "
13 "	4.75	22.56	61.93	40.37	37.30	.924	.071 "	.1633 "	1.53 "	4.95 "
14 "	4.82	23.23	69.72	46.99	40.00	.851	.061 "	.1403 "	1.33 "	5.80 "
15 "	4.85	23.52	79.27	56.75	43.20	.761	.051 "	.1173 "	1.15 "	6.95 "
16 "	4.89	23.91	88.34	64.43	45.80	.700	.044 "	.1012 "	1.00 "	8.1 "
17 "	4.92	24.21	95.64	72.43	48.20	.648	.038 "	.0874 "	.081 "	9.25 "

$\sigma_\infty = 9.24 \times 10^{15}$ observed.

values of $f(\lambda)$ for the various wave-lengths, we may proceed to solve for values of N. The values of N are given in column 10, of each of the above tables, for the three metals above mentioned.

The values of N for silver show no constant variation within the range of wave-lengths from 6×10^{-5} cm. to 15×10^{-5} cm. and judging from the values contained in column 10, we should be justified in taking the average of the values contained therein, which is $N_1 = .37 \times 10^{23}$.

Thus it seems that the effect of dropping the term in the development of Drude's theory, which expresses the effect of the vibrating corpuscles, has had little influence in the values of N over the range of wave-lengths considered.

If it had any effect we should expect the values of N to appear as a function of the wave-length, the number N decreasing as we approach the wave-length corresponding to the free period of the vibrating corpuscle. This free period, which makes its presence known by a heavy absorption band, lies well in toward the blue of the visible spectrum, and hence may be regarded as remote from the regions for which the values of N have been obtained.

If we now turn our attention to the table containing the results for gold, we find that the value of N appears as a function of the wave-length up to a certain value, namely, a wave-length 10×10^{-5} cm., where the number N is $.31 \times 10^{23}$. From the wave-length 10×10^{-5} cm. to 18×10^{-5} cm. the number N does not appear to be a function of the wave-length and as in the case of silver we are justified in taking the average of the values of N in the interval 10×10^{-5} cm. and 18×10^{-5} cm. Thus giving a value for N equal to $.33 \times 10^{23}$.

We here see the effect of neglecting the term which expresses the influence of the vibrating corpuscle, for as we approach the wave-length which corresponds to the free period of the vibrating corpuscle, the number N diminishes. The absorption band which corresponds to the free period of the vibrating corpuscle does not lie as far in the blue region of the visible spectrum as does the absorption band corresponding to that of the silver corpuscle, and we should expect a slight variation of N for diminishing wave-lengths in the neighborhood of the absorption band of gold.

For the case of platinum we have a very irregular series of values. In Table III. the values of N at a wave-length 5.89×10^{-5} cm. is $.92 \times 10^{23}$, it rises to a maximum of 4.4×10^{23} at a wave-length 10×10^{-5} cm. and then decreases to a value $.82 \times 10^{23}$ at a wave-length 17×10^{-5} cm. The most immediate explanation for these irregularities in the values of N is that there may have been considerable experimental error in the determination of n and a, upon which the value of N chiefly depends; and yet it may be that the absorption band which occurs in the visible spectrum was great enough to cause the irregular behavior shown by the results. If values of n are known for longer wave-lengths, the value of N for these wave-lengths would in all probability become a constant as in the case of silver and gold.

Having obtained values of N and r (as a function of λ), it is, of course, possible to calculate σ_λ as a function of the wave-length, by means of the relation

$$\sigma_\lambda = \frac{N}{r_\lambda}.$$

The values of r_λ and N are known from the tables, and the last column in each table reports the corresponding values of σ_λ. In the case of platinum the value $.81 \times 10^{23}$ has been assigned to N, as it appears for the wave-length 17×10^{-5} cm. This value is in all probability too high, for the values of N had not reached a constant value at that wave-length, and at some longer wave-length N would have been a constant, and less than $.81 \times 10^{23}$.

The above relation applied to silver gives a value of 11.4×10^{15} C.G.S. electrostatic units for σ at a wave-length 6×10^{-5} cm. while for a wave-length of 15×10^{-5} cm. the value is 41.2×10^{15} C.G.S. units where the value of N in each case is taken as the previously mentioned average.

A means of obtaining a value for σ_∞ from the values of $f(\lambda)$ given in column 8, of the table for silver is as follows. Plotting the values of $f(\lambda)$ and λ, we obtain a curve the equation of which is of the general type

$$f(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}.$$

The constants are readily determined to be the following: $A = 265.3$,

$B = 128.5$, $C = 343.3$, regarding the unit of wave-length as 10^{-4} cm. The value of $Pf(\lambda)$ for λ_∞ is readily found and when substituted in the equation $\sigma_\infty = N/r_\infty$, gives for σ_∞ the value 61×10^{15}, as compared with the observed value of 60.9×10^{15}; while the agreement is a remarkable one, it only serves to show that the value of N obtained for silver has some claim to correctness. This method of extrapolation is open to objections as all such methods are. They do not always represent the facts of the case.

Returning to the relation $N/r_\lambda = \sigma_\lambda$, we have obtained results for gold and platinum as reported in Tables II. and III. The conductivity for gold at a wave-length 5.89×10^{-5} cm. is 5.5×10^{15} C.G.S. units and it increases to a value 36.8×10^{15} at a wave-length 18×10^{-5} cm., while the observed value for the conductivity for the steady state is 40×10^{15} C.G.S. units. The results are the same nature as reported for silver, the conductivity diminishing for diminishing wave-lengths.

For platinum the value of the conductivity at a wave-length 5.89×10^{-5} cm. is 3.85×10^{15}, and it diminishes to a value 1.7×10^{15} for a wave-length 8×10^{-5} cm. and then increases until it assumes a value 9.25×10^{-15} at a wave-length 17×10^{-5} cm. while the observed value for steady currents is given as 9.24×10^{15} C.G.S. units Judging from the results for gold and silver the values obtained for the conductivity of platinum are much too high, but we have taken a value of N for platinum, which we know has not yet assumed a constant value, independent of the wave-length, and hence the value of σ_λ for platinum would in general be smaller.

Another phase of the question is of interest, which arises from a consideration of a formula derived by Thomson, and previously mentioned.

$$\sigma_\lambda = \frac{Ne^2}{m} \frac{\sin^2 g\frac{t_2}{2}}{\left(g\frac{t_2}{2}\right)^2} \frac{t_2}{2} \qquad (a)$$

This for σ_∞ becomes

$$\sigma_\infty = \frac{Nct_2}{2m}. \qquad (b)$$

Taking the observed value of σ_∞, which for silver is 60.9×10^{15},

and the value given for N as 0.37×10^{23}, we may obtain a value of t_2, the time of duration of free path as 18.4×10^{-15} sec. This value of 18.4×10^{-15} sec. is about ten times greater than the value obtained by Bloch. The value of N obtained by Bloch is about 10^2 times as large as the value obtained in this paper. In other words if we grant the correctness of Thomson's formula and get the value of N, equivalent to the number of atoms per unit volume, or of magnitude 10^{23}, then the value of t_2 is of a higher order of magnitudes than usually given, it becomes 1.84×10^{-14} sec.

The conclusions reached in this paper are briefly:

1. That the effect of the vibrating corpuscle in the visible spectrum of gold, silver and platinum has very little influence upon the optical properties of these metals after we arrive at a wave-length which for silver and gold is beyond 10×10^{-5} cm., and which for platinum is beyond 18×10^{-5} cm.

2. That the value of N obtained by the method of this paper is of the order of magnitude of 10^{22} for silver and gold, and for platinum is less than 10^{23}. The specific values for silver and gold are 0.37×10^{23}, and 0.33×10^{23} respectively, the value for platinum is less than $.81 \times 10^{23}$. The belief that they are correct is based upon the values of the conductivities to which they lead.

The writer wishes to express his indebtedness to Professor Trowbridge, whose aid directed the writer's thoughts along the proper channels.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
November, 1908.

STUDIES IN LUMINESCENCE.

By Edward L. Nichols and Ernest Merritt.

IX. A Spectrophotometric Study of Certain Cases of Kathodo-Luminescence.[1]

THE spectrophotometric study of luminescence has thus far been confined almost entirely to cases of photo-luminescence. It is of interest to inquire whether exciting agents other than light will give luminescence spectra of the same type, and to what extent the laws that have been found to hold in the case of photo-luminescence possess a more general application. The experiments on kathodo-luminescence described in the present paper are of interest chiefly because of the bearing of the results upon these and similar questions. During the course of the work preliminary data have also been obtained with regard to the dependence of kathodo-luminesence upon discharge potential and current strength.

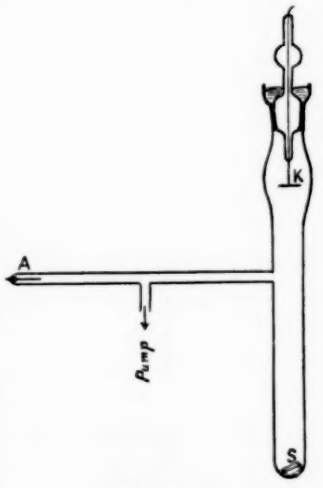

Fig. 73.

The form of vacuum tube used is show in Fig. 73, the substance to be tested being placed at *S*. In order to prevent disturbance from the fluorescence of the glass the specimen in some instances was placed in a metal inclosure which protected the glass from excitation by the kathode rays. The kathode, *K*, was a flat aluminum disk. The fluorescent light was reflected directly into the slit of the Lummer-Brodhun spectrophotometer by a small mirror at the side of the tube. An acetylene flame was used as a comparison

[1] The results contained in this paper were in part presented to the American Physical Society, at the meeting held in Washington, April 24, 1908.

source. Current was furnished by a motor-driven Holtz machine capable of giving a current of 0.6 milliampere. A galvanometer placed in the circuit next to the grounded pole of the machine served to indicate the constancy, or lack of constancy, of the current through the tube. In most cases relative values only of the current have been recorded. The potential difference between the terminals of the tube was measured by a Kelvin electrostatic voltmeter and served as a sensitive indicator of changes in the vacuum.[1]

Considerable difficulty was encountered in keeping the discharge through the tube steady. Sudden changes occurred in the intensity of the fluorescence, even without any corresponding change in current or voltage, due apparently to some erratic cause producing either a deflection of the kathode rays or a change in the intensity of these rays. The most serious of these disturbances were traced to irregular leakage from the wires leading to the Holtz machine. This trouble was made more difficult to control by unfavorable atmospheric conditions, the work being carried on in the spring; but the disturbances due to leakage were largely removed by using large rods in place of wires, and by avoiding sharp corners. It proved advantageous, also, to run the Holtz machine at full speed and to cut down the potential difference between the terminals of the tube by means of resistance in series. The necessary resistance was furnished by a tube containing absolute alcohol. Flickering persisted to some extent even with these precautions, and it was frequently necessary to discard the results of several hours work. The results published were, however, taken under favorable conditions.[2]

It did not prove practicable to maintain the pressure in the tube constant during the time necessary for spectrophotometric measurements throughout the spectrum. Owing to a small leak, or to the development of gas by the discharge, the pressure gradually increased during the course of a series of readings, so that at the end of about an

[1] Since the absolute values of the potential used are of no great significance in the present work we have plotted voltmeter readings rather than the actual values of the potential in the curves accompanying this article. If the potential in volts is desired this may be found by multiplying the voltmeter deflection by 385.

[2] It is probable that the trouble was due to electrostatic charges on the walls of the tube. A larger tube so constructed as to have the kathode at a considerable distance from the walls would probably act more satisfactorily.

hour it was necessary to re-exhaust. In order to determine the luminescence spectrum corresponding to constant pressure conditions the following procedure was adopted: —

While one observer recorded the readings of the voltmeter and galvanometer and, when necessary, operated the pump, the other made settings of the spectrophotometer as rapidly as possible, running back and forth through the spectrum until the whole region had been covered a number of times. Curves were then drawn for each wave-length at which settings had been made by plotting intensities, as measured by the spectrophotometer, against voltmeter readings. A series of such curves is shown at the left in Fig. 74, which refers to a cadmium sulphate preparation. These curves all have the same general shape, and in most cases each curve contains enough points to determine its form with considerable definiteness. After the curves had been drawn as accurately as possible the intensity corresponding to any particular voltmeter reading could be determined by graphical interpolation. The points forming the two curves to the right in Fig. 74 (marked I. V. M. 30 and II. V. M. 17.5) were determined in this way for the voltmeter readings 30 and 17.5 respectively. For these curves the abscissas are wave-lengths expressed in fractions of μ as indicated.

The procedure in the case of our experiments with willemite (Fig. 75) and Sidot blende (Fig. 77) was exactly the same as in the case of cadmium sulphate. A comparison of these three figures shows, however, that while the voltage intensity curves in Figs. 74 and 75 are similar, being in each case concave toward the horizontal axis, the corresponding curves in Fig. 77 are almost exactly straight. The difference results from the fact that in the experiments corresponding to Figs. 74 and 75 the current through the tube diminished rapidly as the voltage increased, whereas in the experiments on Sidot blende (Fig. 77) the conditions as regards leakage were more favorable and the current remained nearly constant. During the observations corresponding to Fig. 77 the deflection of the galvanometer varied from 32 for the highest voltmeter reading (33.8) to 37 for the lowest voltmeter reading (11.7). In the experiments with willemite (Fig. 75) the galvanometer deflection ranged from 7 to 25; and in the experiments with cadmium sulphate the range was from 15 to 31.

The results obtained are plotted in Figs. 74 to 77 and are further discussed below.

$$CdSO_4 + x\,MnSO_4.$$

This substance was prepared by C. W. Waggoner, who has determined its decay curve when excited by the iron spark.[1] Excited by kathode rays it gave an intense yellow fluorescence, with scarcely observable phosphorescence. Close inspection of the powder showed occasional grains which glowed with an orange or

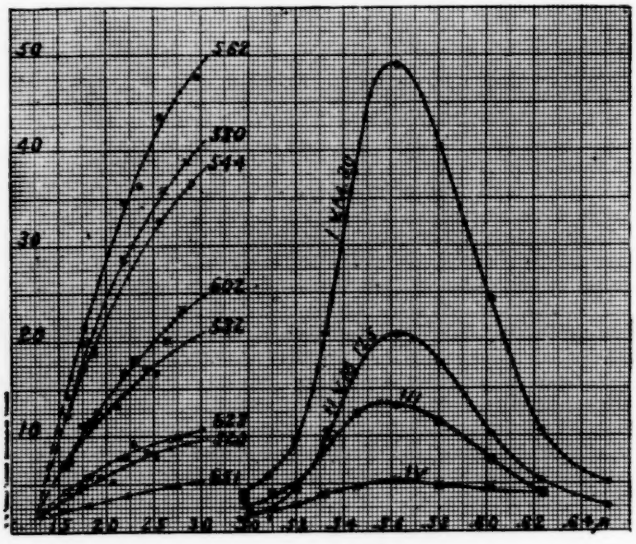

Fig. 74. $CdSO_4 + x\,MnSO_4$.

The curves to the left show the relation between discharge potential and intensity for different wave-lengths. Curves I. and II. show the luminescence spectra for discharge potentials of 11600 volts and 6700 volts respectively. Curves III. and IV. show, to a larger scale, the luminescence spectra excited by ultra-violet light and Röntgen rays respectively.

red light. At the end of our experiments, after the powder had been bombarded by the kathode rays for about ten hours, the surface was found to have acquired a ruddy brown discoloration. Upon standing for several months the powder scarcely responded at all to excitation by the iron spark, but recovered its activity after heating.

[1] PHYS. REV., XXVII., p. 209, 1908.

Inspection of Curves I. and II., Fig. 74, shows that the form of the fluorescence spectrum is independent of the discharge potential, and therefore of the velocity of the cathode rays, since it is clear that one of these curves might be obtained from the other merely by changing the scale.

The fluorescence spectrum was also determined for excitation by the ultra-violet rays in the spectrum of the iron spark, and for excitation by Röntgen rays, the results being shown in Curves III. and IV. respectively. The luminescence excited in both of these rays was weak and the spectrum could be explored only with great difficulty. Curves III. and IV. are drawn to a much larger scale than Curves I. and II. in order that they may be shown in the same figure. In reality the intensity at the maximum of Curve I. is 200 times as great as the corresponding intensity for Curve III., and 500 times as great as the maximum for Curve IV. It will be observed that the maximum of Curve IV. occurs at the same wavelength as in the case of cathodo-luminescence. In the case of photo-luminescence, Curve III., there is an apparent shift toward the shorter waves; we are of the opinion however that this shift is not real but due to errors resulting from the extreme faintness of the spectrum.

The substance continued to glow for a long time when excited by ultra-violet light, but after excitation by kathode rays the phosphorescence died out with great rapidity.

Willemite.

The specimen tested was a part of the same piece that had previously been used in determining the decay of phosphorescence in this substance.[1] No change was observable in the specimen even after prolonged excitation.

While the kathodo-luminescence of willemite was not much more intense than that of $CdSO_4$, the photo-luminescence, excited as before by the ultra-violet rays from an iron spark, was found to be much brighter than that of $CdSO_4$. The luminescence excited by Röntgen rays was, however, only slightly more intense. Curves IV. and V., Fig. 75, which show the spectral distribution for the

[1] Nichols and Merritt, PHYS. REV., XXIII., p. 37, 1906.

photo-luminescence and the Röntgen luminescence respectively, are plotted to a larger scale than the curves for kathodo-luminescence on the same figure, as it would otherwise be impossible to see the details of these curves. The maximum intensity for Curve I. was in reality 120 times as great as that for Curve IV. and 1,500 times as great as for Curve V.

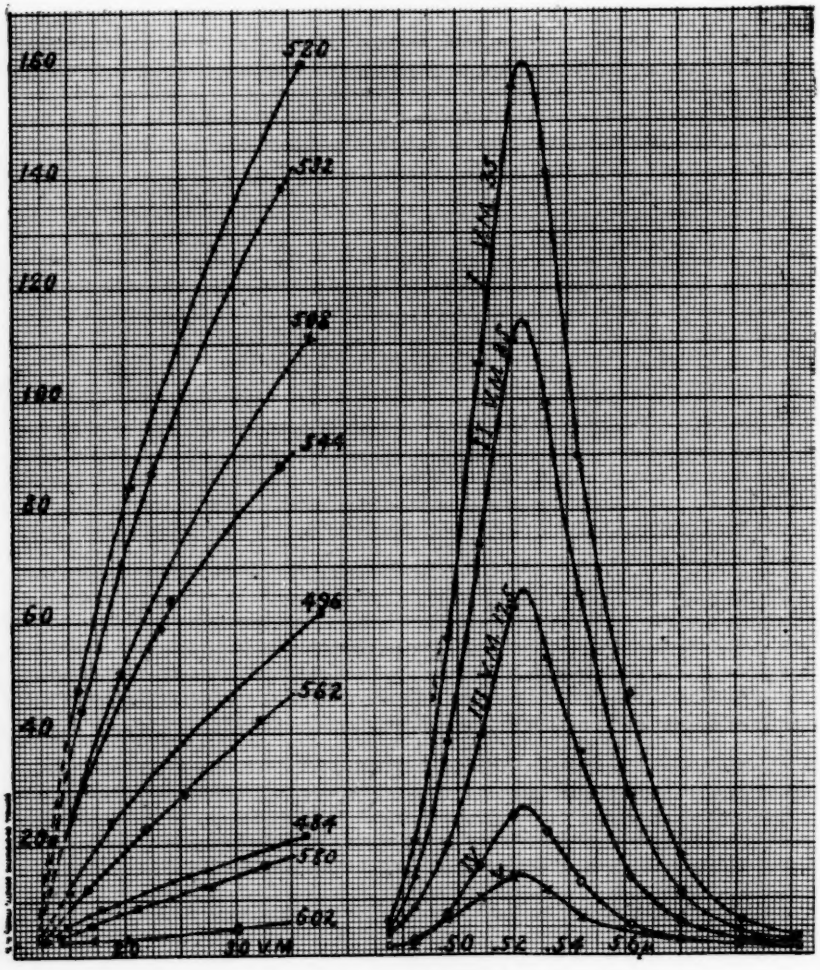

Fig. 75. Willemite.

The curves to the left are voltage-intensity curves for different wave-lengths. Curves I., II., and III., show the luminescence spectra for discharge potentials of 13,500 volts, 9,600 volts, and 6,700 volts respectively. Curves IV. and V. show, to a large scale, the luminescence spectra excited by ultra-violet light and Röntgen rays respectively.

Inspection of the curves of Fig. 75 seems to justify the conclusion that the luminescence spectrum of willemite has the same form whether the exciting agent is ultra-violet light, Röntgen rays, or kathode rays. The form of the spectrum is also seen to be independent of the potential difference between the terminals of the tube, and is therefore independent of the velocity of the kathode rays.

When excited by ultra-violet light the willemite used in these experiments showed bright phosphorescence, whose rate of decay was so slow as to be comparable with that of Sidot blende. But when excited by kathode rays the phosphorescence was faint and disappeared within a few seconds. The intense brilliancy of the kathodo-luminescence during excitation makes this difference especially striking.

Sidot Blende.

The specimens tested were pieces of the same screen that had been used in our previous work on Sidot blende.[1]

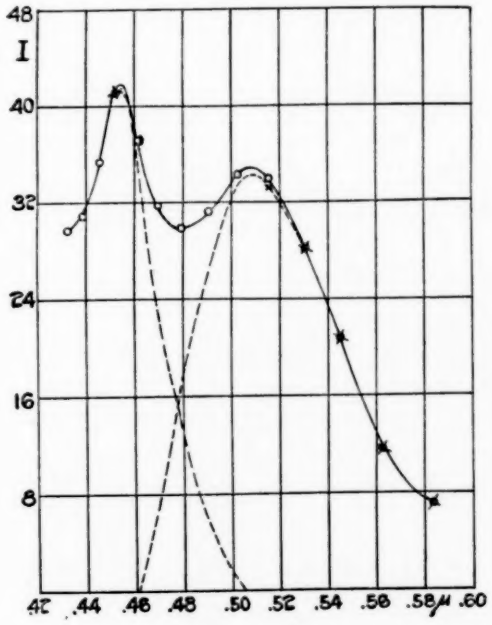

Fig. 76. Sidot Blende.

[1] Nichols and Merritt, Phys. Rev., XXI., p. 247; XXII., p. 279; XXIII., p. 37; XXV., p. 362.

Our first measurements of the kathodo-luminescence of this substance were made in connection with this earlier work, and although the conditions were not so definite as in the case of the more recent investigation, the result of one exploration of the spectrum, shown in Fig. 76, will be of interest for purposes of comparison. Kathode rays were developed in these experiments by connecting the tube used with the secondary of an induction coil, the primary of which was supplied with alternating current. The discharge was made approximately unidirectional by placing a rectifying tube in series. The current was controlled by an alcohol resistance. The current used being smaller than that employed in our more recent work, less difficulty was met with in maintaining constant pressure in the tube. That the pressure was nearly constant during the measurements corresponding to Fig. 76 is shown by the fact that it was possible to run through the spectrum a second time with practically identical results. The points marked by circles in Fig. 76 were determined first, while those marked with crosses were taken afterwards, with no change in the conditions, as a check. It is clear that the spectrum contains two overlapping bands, and the estimated curves for the separate bands are indicated by the dotted lines.

Our more recent measurements with Sidot blende were made by the same method as that used with $CdSO_4$ and willemite. The results are shown in Fig. 77.

When subjected to the relatively intense kathode rays used in these experiments the Sidot blende was found to undergo a rapid change, which was manifested both by a discoloration of the surface and by a diminution in the intensity of the luminescence. In Fig. 77, Curve III. corresponds to the same voltmeter reading as Curve I., but was taken after the substance had been excited continuously for two or three hours. Not only is the intensity much diminished by prolonged excitation but the whole character of the luminescence spectrum is altered.

The band at about 0.51 μ, which appears in Fig. 76 and in Curves I. and II. of Fig. 77, appears to be the same as that excited by light and by Röntgen rays.[1] The band at about 0.455 μ is also excited by Röntgen rays.[2] But this band is either not present in

[1] Nichols and Merritt, PHYS. REV., XXI., p. 247, 1905, Figures 30, 31 and 35.
[2] *Loc. cit.*, Fig. 30.

the spectrum excited by ultra-violet light, or is masked by the other bands present in that spectrum. The luminescence of Sidot blende, as we have previously pointed out, is extremely complex, and the green band at about 0.51 μ is the only one which can be readily isolated.

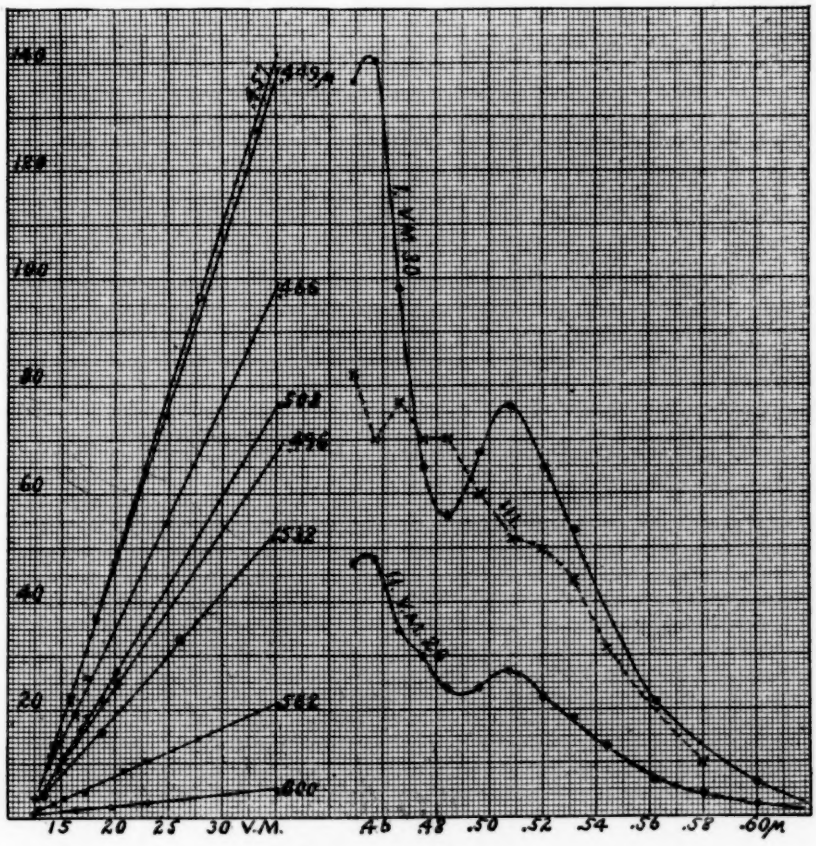

Fig. 77. Sidot Blende.

The curves to the left are voltage-intensity curves for different wave-lengths. Curves I. and II. show the luminescence spectra for discharge potentials of 11600 volts and 7700 volts respectively. Curve III. shows the luminescence spectrum for 11600 volts after the substance had been altered by the action of the kathode rays.

When excited by kathode rays the phosphorescence of Sidot blende is faint and of relatively short duration.

The infra-red rays obtained by interposing a thin sheet of hard

rubber in the path of the rays from an arc diminished the kathode luminescence of Sidot blende only slightly. A long series of settings, alternately with and without infra-red rays, was necessary to make certain that any effect was produced at all. Infra-red rays of the same intensity would have cut down the fluorescence excited by light to one-half its normal intensity.[1]

DEPENDENCE OF KATHODO-LUMINESCENCE UPON CURRENT AND DISCHARGE POTENTIAL.

Since kathodo-luminescence results from the bombardment of the luminescent substance by the kathode rays it is natural to expect a simple relation between the intensity of luminescence and the velocity and number of the kathode ray particles. The experiments here described show that the *form* of the luminescence spectrum is independent of both these factors. The experiments show also that the intensity of luminescence is increased by increasing either the velocity of the rays or the current in the tube. But the conditions of the experiments were unfortunately not such as to permit definite conclusions to be drawn regarding the quantitative relations.

Several attempts were made to determine the relation between intensity and current at a constant discharge potential. The current could be varied through a wide range either by changing the speed of the Holtz machine or by altering a resistance in multiple with the tube. The results make it seem probable that the intensity of luminescence is proportional to the current; but erratic changes in the conditions in the tube, due probably to static charges on the walls, made it impossible to get entirely consistent results. For a time the conditions would apparently remain constant and the intensity was found to vary in the same ratio as the current. But some source of disturbance soon developed, so that the next observations were discordant. Some change in the form of tube or the source of current supply will be necessary before this question can be definitely settled.

Working with the relatively slow kathode rays developed by ultra violet light Lenard has found the intensity of luminescence,

[1] PHYS. REV., XXV., p. 362, 1907.

I, to be related to the discharge potential, V, by the equation

$$I = CQ(V - V_0)$$

where Q is the density of the kathode stream, C a constant, and V_0 a minimum potential below which no luminescence is produced.[1] The existence of a sharply defined lower limit to the potential that is capable of producing luminescence is called in question by Wehnelt,[2] who used in his experiments a much greater density of the kathode rays. On the other hand a definite lower limit to the velocity has been found by Rutherford[3] in the analogous case of luminescence produced by the α rays of radium.

Our own experiments with Sidot blende appear at first glance to confirm the conclusions of Lenard, for the voltage-intensity curves, Fig. 77, are straight lines cutting the horizontal axis at a point corresponding to a voltmeter reading of about 12 divisions. In the case of the experiments with Sidot blende the current in the tube varied only slightly. We are not justified in concluding, however, that the density of the kathode stream was constant, for the fraction of the current that is carried by the kathode rays is known to vary as the vacuum changes, being practically zero at high pressures and nearly 100 per cent. when the pressure is low. In our experiments it was not practicable to determine the amount of this change. It seems scarcely probable, however, that the change was unimportant. If the curves of Fig. 77 could be corrected so as to refer to a constant density of the kathode stream it appears probable that they would differ greatly from the curves plotted.

Conclusions.

In so far as it is possible to generalize on the basis of the small number of experiments here described, the following conclusions appear to be justified.

1. In cases of kathodo-luminescence the distribution of intensity throughout each band of the luminescence spectrum is independent of the discharge potential, and therefore independent of the velocity of the kathode rays.

[1] Lenard, Annalen der Physik, 12, p. 449, 1903.

[2] Wehnelt, Berliner Physikalische Gesellschaft, 5, p. 225, 1903.

[3] Rutherford, Radioactivity, second edition, p. 547.

2. In cases where a band is capable of being excited by light and by Röntgen rays as well as by kathode rays, the form of the band and the position of its maximum are the same for all these modes of excitation.

We have previously shown that the distribution of intensity in any given band is independent of the wave-length of the exciting light.[1] It appears probable, therefore, that the above conclusion may be stated more broadly, and that the distribution of intensity in each band of a luminescence spectrum is independent of the nature of the exciting agent by which the luminescence is produced.

3. Phosphorescence following excitation by kathode rays is less intense and more fleeting than the phosphorescence excited in the same substance by light.

The explanation of this fact is probably to be found in the relatively slight penetrating power of the kathode rays. The excitation is thus confined to a thin layer at the surface of the active substance, while in the case of photo-luminescence the excitation extends to a considerable depth. As pointed out in the preceding paper of this series,[2] the decay of phosphorescence would be more gradual in the latter case.

4. The effect of infra-red rays upon the kathodo-luminescence of Sidot blende during excitation is small compared to the same effect upon the photo-luminescence of this substance and produces a barely observable decrease in the intensity of luminescence.

[1] Nichols and Merritt, PHYS. REV., XVIII., p. 403; XIX., p. 18.
[2] Nichols and Merritt, PHYS. REV., XXVII., p. 373, 1908.

THE EFFECT OF TEMPERATURE ON FLUORESCENCE AND ABSORPTION.

By R. C. Gibbs.

I. Fluorescence and Absorption of Canary Glass at High Temperature.[1]

THE desirability of investigating the effect of change of temperature on the fluorescence and absorption of various specimens of glass was suggested to the writer by Professor Nichols. Few quantitative results have been published showing the nature of such effects. It is not to be expected that, from the results obtained in the case of a single specimen, one could predict the behavior of other specimens though they may be very similar in composition. The following observed phenomena serve, therefore, only as a starting point in this line of investigation.

This paper deals only with the high temperature effect in the case of canary glass. By high temperature is meant such a temperature to which we may heat the specimen in question without materially changing its hardness.

The glass used in these experiments was obtained through the kindness of Professor Bancroft from the Geophysical Laboratory at Washington. A statement from this laboratory shows that the glass was prepared according to the following formula: SiO_2 70 per cent., K_2O 24 per cent., CaO 6 per cent. to which 2.5 to 3.0 per cent. sodium uranate is added.

The glass was ground approximately into the shape of a rectangular plinth, 2.88 cm. by 3.37 cm. with an average thickness of .74 cm. All faces were polished except the two smallest.

In order to eliminate transmitted light as much as possible from the observations of fluorescence, the glass was mounted in an iron case which completely covered the unpolished faces of the glass,

[1] A summary of the work described in this paper was presented to the Physical Society at the Hanover meeting, June 30, 1908.

and left exposed a rectangular surface 2.66 cm. by 1.88 cm. on the two largest faces and a similar surface .5 cm. by 1.88 cm. on the other two faces. These exposed surfaces were located about equally distant from the parallel edges of the glass. In this way the edges of the glass which contained small nicks due to grinding were covered, thus eliminating irregular reflection. The plan, front elevation and end elevation of the case are shown in Fig. 1. The heavy broken lines indicate the position of the glass.

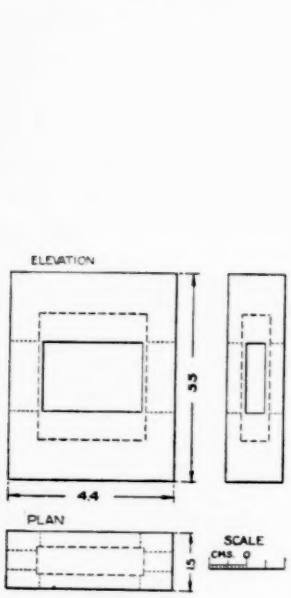

Fig. 1.

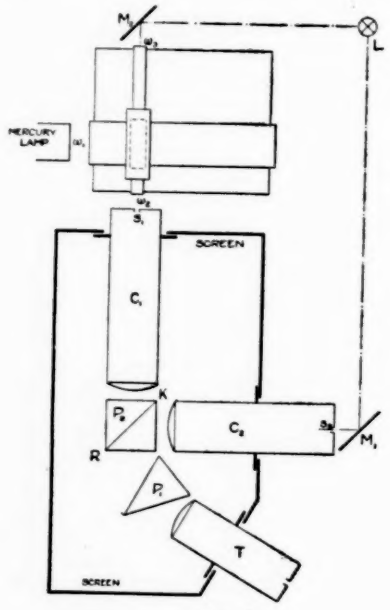

Fig. 2.

The case is made of six pieces of iron fastened together with screws and so constructed that the glass can be taken out after removing one of these pieces.

The framework of the furnace used for heating the specimen was made from asbestos cement roofing plates fastened together by iron rods in the shape of a box about 11 cm. by 9 cm. by 15 cm. The iron case was placed inside the furnace box in a position indicated in Fig. 2.

Four rectangular tubes made of blackened tin were fastened to the iron case so that the axes of the tubes were respectively perpen-

dicular to the four exposed surfaces of the glass and approximately at their centers. The cross section of each tube was similar in shape to and slightly larger than the surface of the glass which it covered. The outer ends of these tubes projected a short distance through the sides of the furnace box.

Mica windows were carefully fitted into grooves made in these tubes near their outer ends. The object of making four openings from the glass to the outside of the furnace will appear later when the method of measuring the fluorescence and absorption are described.

To secure heating an electric current was passed through iron wire wound in four coils on porcelain tubes which were supported inside the furnace box by iron rods. The iron case and rectangular tubes were carefully wrapped with several thicknesses of asbestos paper in order to secure as uniform a temperature in all parts of the glass as possible. To prevent loss of heat and also unequal heating all imperfect joints and cracks around tubes, wires and rods were for the most part sealed up with asbestos cement paste. The whole was carefully wrapped with several coatings of asbestos paper.

The temperature was measured by means of an iron-constantan thermo-junction. A potentiometer was used to measure the E.M.F. One junction was placed in ice and the other was placed in a groove in the iron case so that it came in contact with one face of the glass just above the opening through which the fluorescence was observed. The thermo-junction was calibrated by placing one junction in ice and the other very close to the bulb of a high temperature mercury thermometer and inserting them in a porcelain tube. Each end of the tube was tightly closed and the tube heated by means of an electric resistance properly insulated. The thermometer was read for both rising and falling temperatures at equally spaced intervals of E.M.F. The average of these readings was then plotted against the E.M.F. and from this curve the E.M.F. for any desired temperature was obtained. It was deemed unnecessary to apply the usual corrections to the readings of the thermometer because the phenomena observed did not show a measurable change for such small differences of temperature.

The light used for exciting fluorescence was obtained from the

arc of a mercury lamp. The lamp was placed in front of the mica window W_1 (Fig. 2) about 3 cm. from the surface of the glass. An ammeter was placed in series with the lamp and its reading frequently observed in order to guard against combining results obtained with a variable intensity of excitation. Since the lamp had to be inclined to start the arc, it was securely fastened by a hinge arrangement so that it would always come back to the same position. It may here be noted that the ammeter showed an almost constant reading during the observations from which the following curves were obtained. The face of the glass directed away from the lamp was left uncovered in order to prevent the reflection of the light from the lamp as much as possible. Such a reflection might cause the transmission to the slit of a considerable portion of the exciting light.

The fluorescence was observed through the mica window W_2 (Fig. 2) and its intensity for the various wave lengths was measured by means of a Lummer and Brodhun spectro-photometer shown in Fig. 2. The prism P_1 was adjusted for minimum deviation of the sodium line. The telescope T was clamped in such a position as to afford freedom of range over the entire spectrum by means of the vernier screw. The collimator C_2 was adjusted until the image of the sodium line from the slit S_2 coincided with that from S_1.

The more important lines of hydrogen, helium, sodium and neon were used to obtain a calibration curve from which all vernier readings were reduced to wave lengths.

The fluorescent light fell upon slit S_1 (Fig. 2) which was kept at a constant width of one millimeter and its intensity was measured by adjusting the width of slit S_2 until the light reflected from RK showed the same intensity as the light transmitted through the upper and lower parts of the prism P_2 from S_1. The source of light which illuminated slit S_2 was an acetylene flame L from which light was diffusely reflected by a block of magnesium carbonate M_1, placed at an angle of 45° with the axis of the collimator C_2. In making settings of slit S_2 care was taken to avoid lost motion of the micrometer screw. A water pressure gauge connected to the source of the acetylene supply served to indicate possible changes in intensity of the comparison light. The acetylene pressure was constant when the following results were obtained.

Light from L was reflected by means of a thin glass mirror M_2, placed at an angle of about 45° with the axis of collimator C_1, so as to pass through mica window W_3 and fall upon the glass. The intensity of the light transmitted through the 2.88 cm. of glass was measured in the same way as that of the fluorescent light.

The glass mirror was used instead of a magnesium carbonate block because of the large absorbing power of the glass. It was found, however, upon removing the glass in order to measure the intensity of the incident light that a diffuse reflector was desirable. The glass mirror was then replaced by a block of magnesium carbonate and the number of acetylene flames increased from one to five. The transmitted and incident lights were then measured for the various wave lengths at room temperature. From these results together with those obtained with the mirror for transmission at room temperature, the incident light in the case of the mirror was computed for the various wave-lengths. This was done by assuming that the ratios of the intensity of incident light to that of the transmitted light were equal in the two cases.

When observing fluorescence window W_3 was covered; and the circuit through the mercury lamp was broken when measuring transmission.

Screens were used for three purposes:

1. To prevent any light reaching the prisms except through the collimators. To this end the spectro-photometer was completely boxed in by pasteboard covered with black paper. The plan view of this box is shown in Fig. 2.

2. To prevent any light entering slit S_1 or slit S_2 except such as came through window W_2 or was reflected by M_1 from L. To insure this window W_2 was covered and M_1 screened from L and screens were then erected at various places until no light could be distinguished through the telescope when all lights to be used were going and when the slits of both collimators were wide open.

3. To render the room as dark as possible in order to bring the eye to a sensitive condition for observation. For reading verniers small telephone lamps were used with their direct light screened from the eye.

After adjusting the position of the spectro photometer to secure

uniform intensity of field for both the fluorescence and transmission, they were carefully measured at room temperature. Then a trial was made to find the effect of heating. Rough measurements showed considerable decrease in fluorescent intensity for various parts of the spectrum for a change of about 200° in temperature. Twenty-four hours later the observations for room temperature were repeated with the result that a somewhat greater intensity of fluorescence prevailed throughout the main band of the fluorescence spectrum than in the previous room temperature readings. Heating then to 300° showed again a considerable reduction in fluorescent intensity. A third set of readings for room temperature, taken twenty-four hours after the second heating, did not show as great an intensity as the second set, but greater than the first set, indicating that the hysteresis effect became less for repeated heatings. After standing a few days the readings showed a nearer return to those of the first set.

About three days after the second heating the intensities of fluorescence and transmission at room temperature were again measured throughout the fluorescent spectrum. Heat was then applied and in order to save time and finish the observation in one day — for it was desirable to heat the glass very slowly — the readings for the fluorescence were taken while the temperature was changing. The reading for temperature was taken before each observation for fluorescence. In this way five settings for each wave-length were secured over a range of temperature from about 25° to about 360°. The intensity of fluorescence was then plotted as a function of the temperature for each wave-length observed, thirty-four in all. From these curves the fluorescence for any temperature between the above limits could be read off even though no setting had been made at that particular temperature.

Observations for transmission were made in a similar manner except that the temperature was kept practically constant each time for all parts of the spectrum. This procedure was followed in order that the glass might be under excitation during the greater portion of the change in temperature.

Over two weeks later a similar but independent set of observations were made over the same range of temperature. The results of

the two sets of observations agreed in all important details except that the fluorescence at room temperature was somewhat less in the second set and showed a close agreement with the observations made before heating. This again shows that although the fluorescence at room temperature is greater shortly after heating than before, yet it gradually returns to its normal value if allowed to stand.

The following tables and curves show the results obtained from the last set of observations. The questions of the hysteresis previously mentioned, the gradual return to normal fluorescence, and the effects due to different methods of excitation and observation do not directly concern this paper and are therefore left for more systematic investigation.

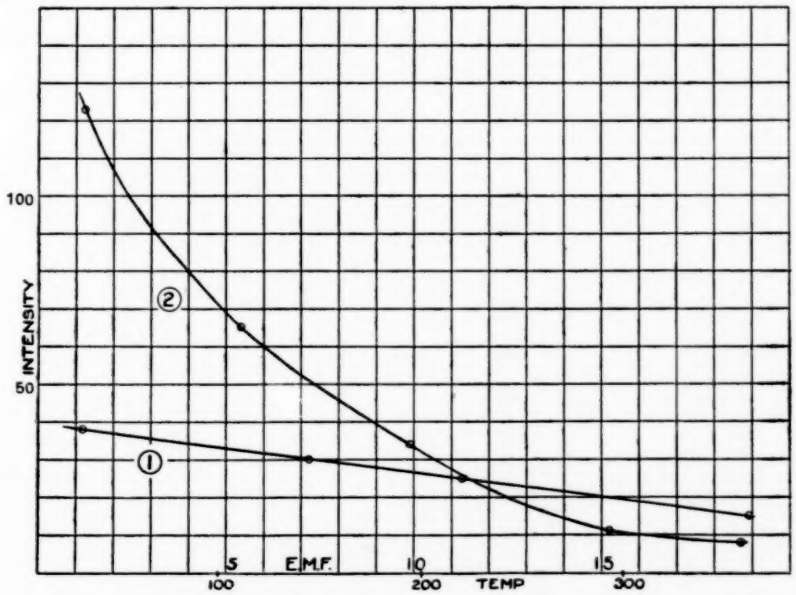

Fig. 3. Curve 1, Intensity of transmission for $\lambda = .509\,\mu$. Curve 2,[1] Observed intensity of fluorescence for $\lambda = .530\,\mu$.

Table I. shows the intensity of incident light from M_2 (Fig. 2) together with the observed transmitted intensities and temperature read-

[1] An error was made in plotting two of the points for this curve. The points having ordinates of 123 and 65 should have been placed with their present abscissa at 133 and 75 respectively, as reference to table II will indicate. The incorrect curve, however, shows the general nature of this type of curves.

TABLE I.

λ	I	T	$E/10^2$	T	$E/10^2$	T	$E/10^2$	T	$E/10^2$
.417μ	404	24	11.7	20	70	15	113		189
.420	388	22		19	71	16			
.423	370	20		18		13			
.426	296	16		13		13		11	
.429	267	13		11		11		11	
.432	278	12		10		10		10	
.436	342	12		10		9		9	
.439	370	12		10		9		9	
.443	444	12		10		9		9	
.446	407	11		9		8		9	
.450	370	11		8		8		8	
.454	370	11		9		9		8	
.458	404	12		9		9		7	
.462	404	12		10		9		7	
.466	414	13		11		10		7	
.471	448	15		13	72	10		8	
.475	455	16		13		10		8	
.481	487	15		12		12		8	
.485	432	14		12		12		8	
.491	455	16		14		12		8	
.497	502	19		16		15		10	
.502	592	24		18		17		10	
.509	703	38		30		25		15	
.516	714	54		45		39		25	
.522	707	65		56		49		50	
.530	711	68		60		56		56	
.537	721	74		69		66		64	
.545	729	78		76		75		74	
.552	776	86		85		85		84	
.560	820	94		94		93	114	94	
.569	864	99		99		99		99	
.578	835	101.5		101		101		101	
.587	836	104		104		104		104	
.597	831	106	11.9	106	72	106	114	106	189

λ = wave-length. I = incident light. T = transmitted light.
E = E.M.F. in micro-volts.

ings (in terms of E.M.F.) for the various wave-lengths. Table II. shows observed values of fluorescence intensity and temperature (E.M.F.) for the same wave lengths.

From the data given in Tables I. and II. curves were drawn for each wave-length showing the effects of temperature upon both the transmission and fluorescence intensities. In Fig. 3 are shown two

TABLE II.

λ	F_0	$E/10^2$	F_0	$E/10^2$	F_0	$E/10^2$	F_0	$E/10^2$	F_0	$E/10^2$
.417 μ	43	12	29	23	17	74	11	122		
.420	39		27	24	16	75	11	123		
.423	31		21	25	15	76	10	124		
.426	27		20	26	14	77	12	125	10	181
.429	27		23	27	15	78	15	127	14	182
.432	27		27	28	24	79	24	128	19	
.436	26		24	32	20	80	19	129	17	
.439	22		19	33	18	81	15	130	12	183
.443	15		15	34	11	82	9	132	9	
.446	14		11	35	9	83	8	133	8	
.450	14		10	36	8	84	7	134	6	
.454	12		10	37	8	85	6	135	6	
.458	12		9	38	8	86	6	136	6	
.462	12		8	39	8	86	6	137	5	184
.466	12		8	40	8	87	6	138	5	
.471	12		8	41	8	88	6	139	5	
.475	12		9	42	8	89	6	140	5	
.481	12		9	43	9	90	7	142	5	
.485	14		11	44	9	91	7	143	5	
.491	21		17	45	12	92	8	144	5	
.497	35		26	46	17	94	7	145	5	
.502	67	12.5	45	47	20	95	7	146	6	
.509	90		54	48	27	96	7	147	6	
.516	114		62	50	31	97	8	148	6	185
.522	128		68	52	32	98	8	150	6	
.530	133		75	54	34	99	11	152	8	186
.537	142		95	57	68	101	57	153	51	
.545	136		99	58	86	104	65	155	60	
.552	111		84	60	80	106	61	157	53	187
.560	79		53	61	48	107	38	159	36	
.569	62		40	62	37	108	34	161	29	188
.578	43		33	63	35	109	29	168	26	
.587	29		22	64	25	110	26	169	24	
.597	19	12.5	19	65	19	111	19	170	17	189

λ = wave-length. F_0 = observed fluorescence. E = E.M.F. in micro-volts.

of these curves. Curve 1 represents the transmission intensity for $\lambda = .509\,\mu$, curve 2 the observed fluorescence intensity for $\lambda = .530\,\mu$.

From the 68 curves similar to those just mentioned the intensities of transmission and fluorescence were read off for seven different temperatures ranging from 25° C. to 350° C.[1] The change in transmitted intensity due to temperature was rather small in some parts

[1] See Table III.

of the spectrum. Consequently only the intensities at 25° and 350° as shown in Fig. 4 have been plotted. The curves for the intermediate temperatures would lie between these two curves and the data indicate that for all temperature changes the change in absorption was in the same direction for all wave-lengths.

The curves in Fig. 4 show that the absorption band becomes somewhat wider on heating and that the absorption for nearly all wave lengths for which the glass shows fluorescence [1] increases when the temperature is raised.

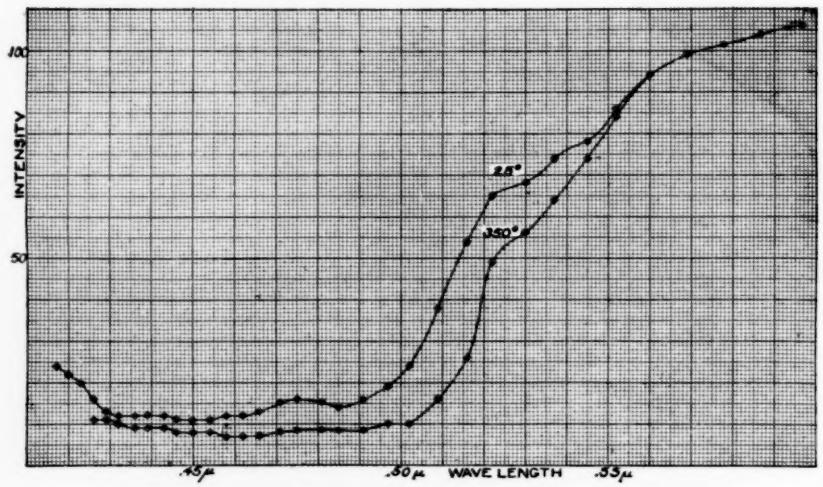

Fig. 4. Showing effect of temperature on transmission intensity.

Since a portion of the fluorescent light is absorbed by the glass before reaching the slit of the collimator, it is necessary to apply a correction to the observed[2] fluorescence to find the typical fluorescence spectrum. Before explaining how this correction was made, it is necessary to define the term coefficient of absorption as it was used in this connection and also to derive the expression from which the coefficient of absorption was computed.

The following is a derivation of what is known as Lambert's law of absorption.

Let I = intensity of incident light.

[1] See Fig. 8.

[2] See Table III.

T = intensity of this light after it has traversed a substance of thickness x.

$-dI_x$ = amount of change in intensity of light when transmitted through thickness dx. Then it is evident that dI_x is proportional to both I_x and dx, *i. e.*, $-dI_x = \beta I_x dx$ wherein β may be defined as the coefficient of absorption of the substance for any single wave-length of light.

Therefore,

$$\frac{dI_x}{I_x} = -\beta dx$$

$$\log I_x \Big]_I^T = -\beta x \Big]_0^x$$

$$\log \frac{T}{I} = -\beta x$$

$$\frac{T}{I} = e^{-\beta x}$$

$$T = I \cdot e^{-\beta x}$$

$$\beta = \frac{\log I - \log T}{x}$$

The values of β for the various wave-lengths and temperatures [1] were computed according to the above formula wherein x had the value of 2.88 cm.

If S is any point in the slit (Fig. 5), the source of light reaching this point consists of a frustum of a cone in the fluorescent glass which, if completed, would have this point for its vertex. The solid angle of the completed cone is fixed by the size of the object glass of the collimator. If any of the rays of light in this cone are intercepted by the sides of the glass the loss will be counterbalanced by total internal reflection.[2]

Fig. 5.

If there were no absorption the amount of light which would reach the point S (Fig. 6) from a small frustum of a cone of height dx and located x distance from S would be $F_e dx$ where F_e is a function of the wave-length. This amount is independent [2] of x because

[1] See Table III.

[2] Wick, Frances G., PHYS. REV., Vol. 24, No. 4, 1907.

the area of the middle section of this small frustum varies as x^2 and the intensity of light reaching S from each infinitesimal portion of this frustum varies as $1/x^2$.

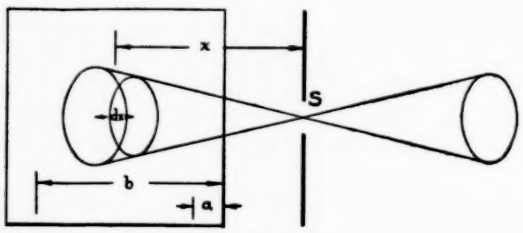

Fig. 6.

Since there is absorption, the light di which reaches S from the small frustum is expressed in the following equation:

$$di = F_c dx e^{-\beta x}.$$

The total light reaching S from the whole frustum is

$$i = F_c \int_a^b e^{-\beta x} dx,$$

where $b - a$ equals the height of the frustum of the cone under excitation. Therefore

$$i = F_c \left(\frac{\varepsilon^{-\beta a} - \varepsilon^{-\beta b}}{\beta} \right),$$

and

$$F_c = \frac{\beta i}{\varepsilon^{-\beta a} - \varepsilon^{-\beta b}}.$$

Although i represents the amount of light reaching an infinitesimal part of the slit, yet the total light reaching the slit is proportional to i, *i. e.*, $i = KF_0$.
Therefore

$$F_c = \frac{K\beta F_0}{\varepsilon^{-\beta a} - e^{-\beta b}},$$

or omitting the constant K in as much, if known, it would affect only the scale to which the results might be plotted, we have

$$F_c = \frac{\beta F_0}{\varepsilon^{-\beta a} - \varepsilon^{-\beta b}},$$

wherein F_0 represents the observed fluorescence intensity, F_c the fluorescence corrected for absorption, β the coefficient of absorption, a and b the distances as shown in Fig. 6. The numerical value for a is .11 cm. and for b 2.77 cm.

TABLE III.[1]

t	I	T	β	F_0	F_c
25	714	54	.896	114	124.2
75		50	.923	79	88.2
125		47	.945	52	59.5
175		43	.976	32	37.5
225		39	1.009	18	21.8
275		34	1.057	10	12.7
350	714	26	1.150	6	8.2

For wave-length = .516 μ. t = temperature. I = incident light. T = transmitted light (obtained from curves similar to (1) Fig. 3). β = coefficient of absorption. F_0 = observed fluorescence (obtained from curves similar to (2) Fig. 3). F_c = fluorescence corrected for absorption.

Table III. shows a few of the values of F_c computed according to the above formula.

The effect on the intensity of fluorescence of correcting for absorption is very slight as the curves in Fig. 7 indicate. The most important result is a small shift of the band toward the violet.

The typical fluorescence curves in Fig. 8 show for the lower temperatures a rather large and somewhat irregular band in the yellow green portion of the spectrum and a smaller one in the region of the violet. The larger band shows considerable change in size and shape at the higher temperatures. The maximum for the band shifts slightly toward the red with rising temperature and the fluorescence between .49 μ and .55 μ is practically destroyed at 350°. The fluorescence in the region of .49 μ remained practically constant during the change in temperature. The intensity of light at .417 μ became considerably smaller with rise of temperature, yet too much dependence should not be placed upon the absolute values of results obtained at this region of the spectrum because of the faintness of the light.

The persistence of the fluorescence at the .545 μ together with its

[1] This table serves as a specimen of indirectly observed and computed values which were obtained for the 34 wave-lengths measured.

rather rapid falling off at the higher temperatures on either side of this wave-length, made it seem desirable to investigate whether the glass was in some way transmitting light from the green line ($\lambda = .546\ \mu$) of the mercury arc, although no sudden rise in intensity, often found in similar cases, had been observed in the immediate neighborhood of this wave-length.

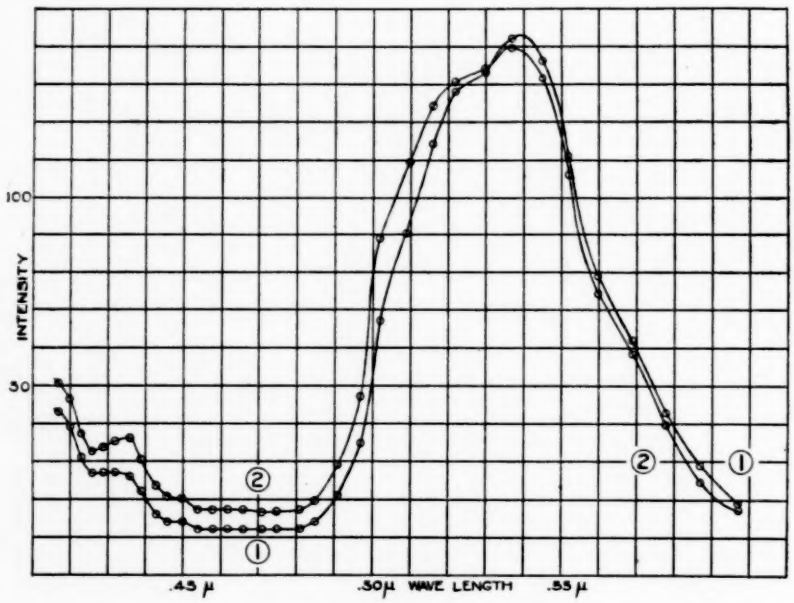

Fig. 7. Fluorescence at 25°. Curve 1, Observed fluorescence. Curve 2,[1] Fluorescence corrected for absorption.

To do this a methyl violet glass which completely absorbed the light from both the bright green and yellow lines of the mercury arc was placed between the lamp and the fluorescent glass and the fluorescence for this part of the spectrum measured, *i. e.*, from .5 μ to .6 μ. Although the ordinates of the curve thus obtained were somewhat less than those of the curve obtained without the violet glass, which was probably due to the absorption of the exciting light, yet the ratio of the ordinates in the two curves was practically constant. Therefore it is evident that the amount of light

[1] A slight error was made in plotting one of the points for this curve. The point having an abscissa of .51 μ should have been placed with the same ordinate at .509 μ.

from the green line of the mercury arc transmitted by the glass is exceedingly small.

The changes in the shape of the curves (Fig. 8) in the region from .5 μ to .53 μ serve to indicate that the main fluorescence band is made up of two, if not three, bands very close together. How much these bands may overlap is difficult to say, but the changes due to temperature may be explained by assuming that one of these bands located at about .545 μ is very slightly affected by heating

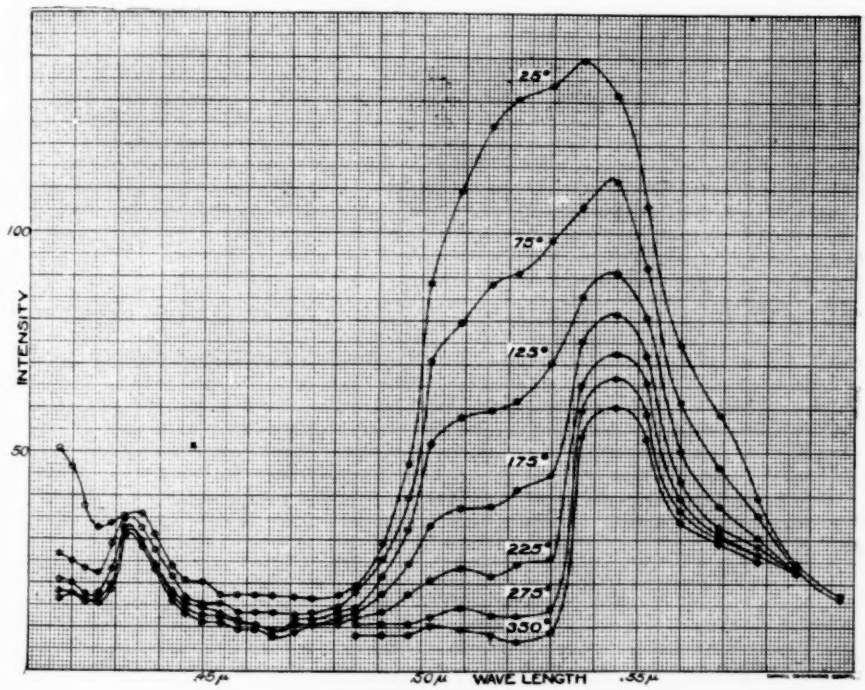

Fig. 8. Fluorescence corrected for absorption.

and the other two, if there are two, have their intensity practically destroyed by a temperature of 350°. If we should assume further that the intensity of the band at .545 μ was slightly increased by a rise in temperature and that its recovery did not keep pace with the fall of temperature on cooling it is possible, in the first place, to account for the constant value of the fluorescence with rising temperature in the region of .59 μ and in the second place to explain

the hysteresis effect previously mentioned. This is, however, only a conjecture which further investigation may show to be false.

The band which shows a maximum at about .433 μ may be due to the transmission of light from the mercury arc which has a bright line at about that wave-length. In as much as that line is probably very effective in exciting fluorescence this possibility could not be tested as in the case of the green line. That this is not the only explanation for this band is evidenced by the fact that the intensity at the maximum increases slightly with a rise of temperature whereas the absorption for that wave-length increases.

SUMMARY.

The essential results of these observations may be stated as follows:

1. Increase of temperature produces an increase in the absorption in the greater part of the spectrum measured and a widening of the absorption band.

2. There is a large and irregular shaped fluorescence band in the yellow-green portion of the spectrum and a smaller band in the violet.

3. The typical fluorescence for the larger band shows for increasing temperature a slight shift in its maximum toward the red.

4. The changes in the typical fluorescence when the glass is heated seem to indicate that the yellow-green band consists of two or more overlapping bands which are affected differently by a rise of temperature.

5. The glass shows a somewhat higher intensity of fluorescence, especially in the region of the main band, immediately after cooling than before heating. It then gradually returns to its normal value

The author wishes to express his gratitude to Professors Nichols, Merritt and Shearer for their many helpful suggestions.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE RADIATION FROM NEUTRAL ELECTRIC SYSTEMS.

BY DANIEL F. COMSTOCK.

THE present paper forms part of an attempt to determine what properties of matter can be explained by the single assumption that the atoms are electric systems.

It seems probable that many of the special assumptions which have been made with regard to atomic structure are "indifferent hypotheses" and, therefore, "cancel out" of the final result. If this be the case such special assumptions lie as obstacles in the path of ultimate explanation, notwithstanding the fact that they may greatly aid the imagination at intermediate stages.

The electric systems here considered are rigid and do not rotate but they may have any distribution of charge whatever consistent with the word "neutral" which here means that there is to be in the system as much positive as negative electricity.

It can be shown that such rigid "neutral" electric systems in general radiate energy when in random motion. It has long been known that a moving point charge, the velocity of which is not too great, radiates energy at a rate proportional to the square of the charge and to the square of the acceleration. If two equal point charges of opposite sign are rigidly bound together at a finite distance from one another the wave of radiation from one interferes with the wave from the other but does not completely neutralize it.

This failure to neutralize follows from the fact that the two charges are at different distances from the point P at which the radiation is to be measured, and hence the two waves which reach P together do not leave their respective charges at the same time and hence, if the acceleration of the system is changing with the time, the amplitudes of the two waves will not be the same and they cannot therefore completely annul each other.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

The effect is seen to be one of a higher order than that of a single point charge. Indeed, as has been hinted, the resulting wave amplitude depends on the *rate of change of the acceleration* and not on the acceleration itself.

The rate of energy radiation from this system will therefore be proportional to the *square* of the rate of change of the acceleration.

By means of considerations similar to those outlined above, it can be shown that the amplitude of the radiation wave from *any* neutral, rigid electric system involves a series of terms of the form

$$\frac{c_1}{V}\frac{da}{dt}, \quad \frac{c_2}{V^2}\frac{d^2a}{dt^2}, \quad \frac{c_3}{V^3}\frac{d^3a}{dt^3}, \quad \text{etc.},$$

where a is the acceleration of the rigid system, V is the velocity of light and c_1, c_2, c_3, etc., are constants depending on the structure of the system.

The nature of the constants c_1, c_2, c_3, etc., is of considerable interest. Analysis shows that the constant c_1 depends on the *electric moment* of the system. The term "electric moment" is here used in the ordinary sense in which magnetic moment is so commonly employed.

The constant c_2 is somewhat similar to a moment of inertia in which electric charge elements take the place of mass elements. Its value is of most importance when the moment of the system is zero, *i. e.*, when the constant $c_1 = 0$. When this condition holds it can be shown that c_2 is entirely independent of the axes chosen and is therefore a fundamental constant of the system.

It can be shown that this term (the one involving c_2) is probably the prevailing one in the radiation from real atoms if they are in reality pure electric systems. This makes the amplitude of the radiation wave proportional to

$$\frac{d^2a}{dt^2}$$

and *the energy radiated from the atom per second proportional to*

$$\left(\frac{d^2a}{dt^2}\right)^2.$$

It is interesting to note that in terms of space and velocity the above expression involves the eighth power of the latter. This is the fourth power of the absolute temperature and suggests harmony with Stefan's law.

It can also be shown on this basis that if a number of atoms have a similar geometrical structure and move in the same way, the rate of energy radiation from each will be proportional to its electromagnetic mass and to the fifth power of any linear dimension.

On the Radioactive Deposits from Actinium.[1]

By Professor J. C. McLennan.

In the course of some experiments which Mr. W. T. Kennedy has been making at Toronto, during the past few months, he has found a marked similarity in the active deposits obtained on positively and negatively charged electrodes placed within an air-tight vessel and subjected to the influence of the active emanation issuing from a sample of actinium.

In his experiments the electrodes consisted of two small circular brass discs provided with guard rings of the same metal and placed parallel to each other at a distance of 2 mm. apart. The discs during an exposure were placed with their planes vertical and directly over an open metal tube 1.5 cm. in diameter with the edges of the guard rings almost in contact with the edges of the upper end of the tube. The salt used was carried in a small tray which could slide freely up and down the tube and by means of a clamp be supported at any required distance from the discs.

In carrying out a set of experiments on the effect of varying the pressure of the air in the vessel containing the discs and the salt, it was found at high pressures that the active deposit appeared almost entirely on the negative electrode. As the pressure was decreased, however, the active deposits on both electrodes increased and ultimately at certain definite pressures, which were different for the two electrodes, reached maximum values. When the pressures were still further lowered, the amounts of the deposit received on both electrodes rapidly decreased and finally approached equality. Curves showing the active deposits obtained on each of the electrodes in a particular set of experiments in air at different pressures are given in Fig. 1, and from them it will be seen that in the region of lowest pressures the deposits on the negative electrode were but slightly greater than those obtained on the positive at corresponding pressures. At the lowest pressure investigated the deposit on the negative electrode was only about 3 per cent. greater than that obtained on the positive plate.

In this set of measurements the salt was placed at a distance of 1 cm. below the plates, and a potential difference of 250 volts was applied to the two electrodes during each of the exposures.

Under these conditions the maximum activity was obtained on the negative electrode at a pressure of about 8 cms. of mercury while the maximum deposit on the positive electrode was not obtained until the pressure was reduced to 1 cm. of mercury. It will be seen that in this case the maximum activity obtained on the negative electrode was about 2.75 times the maximum activity obtained on the positive terminal.

[1] Abstract of a paper presented at the Baltimore meeting the of Physical Society, December 28–31, 1908.

In a second set of measurements with air the electrodes were kept at a fixed distance apart, 2 mm., and the salt was placed at a series of different distances below the exposed plates. Such observations were taken in air at pressures both above and below those corresponding to the maximum deposits on the two electrodes and in every case it was found that as the distance between the plates and the salt was increased the intensity of the active deposits obtained on both plates rapidly decreased.

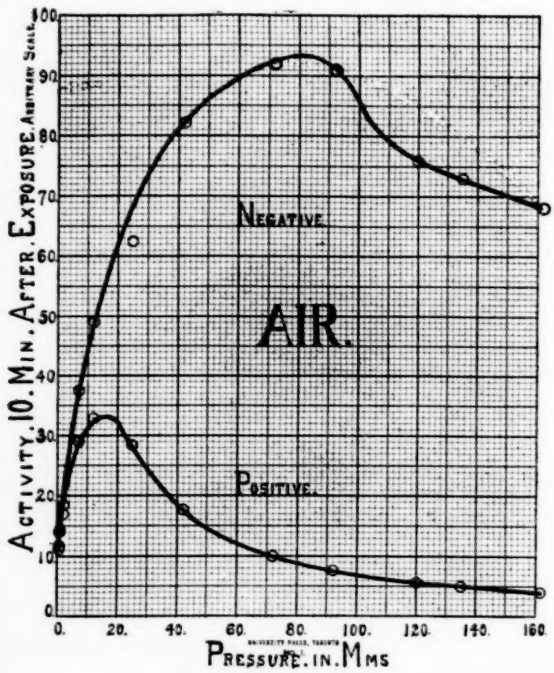

Fig. 1.

Exposures were also made in carbon dioxide and curves drawn from the results obtained with this gas are shown in Fig. 2. From them it will be seen that the maximum activities on the two electrodes were obtained at slightly lower pressures than in air, and that the maximum activities obtained on both electrodes in this gas were somewhat less than those obtained on the same two electrodes in air.

From the rapid decrease in the active deposits which were obtained in both gases at the lower pressures it seems probable that with the arrangement of apparatus used and with the relative distances between the various parts adopted both electrodes would show, at most, very small activities if the exposures were made in high vacua.

In all the experiments at the various pressures in the two gases the discs were exposed for two hours to the action of the emanation from the actinium before being removed from the exposing vessel for measurement and the various readings from which the curves shown in Figs. 1 and 2 were drawn correspond to the intensities of the activity of the deposit on the discs ten minutes after the exposures ceased. The actinium salt used was obtained from the Chininfabrik at Braunschweig, and the active deposits on both electrodes were found to have a decay period of approximately 39 minutes.

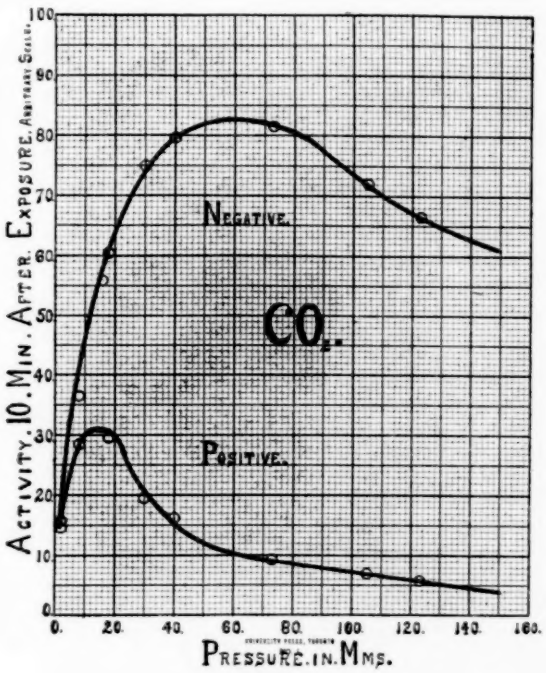

Fig. 2.

As to an interpretation of the results it would seem since the active deposit at very low pressures is approximately equally divided between the two electrodes, that the active deposit particles at these pressures are to a great extent uncharged. From the initial increase in the amount of the deposits obtained on the two electrodes it would seem too that the active deposit particles become charged in their passage through the gas and that on becoming charged their rate of diffusion is increased. The difference in the amounts of the deposits obtained on the two electrodes could on this hypothesis be ascribed to differences in the rate of diffusion of the oppositely charged active deposit particles.

The assumption by which the active deposits obtained on the electrodes are attributed to diffusion of the charged active deposit particles rather than to a diffusion of the emanation under the various circumstances cited, seems warrantable in view of the short decay period of the emanation.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
February 6, 1909.

THE RESISTANCE OF CERTAIN ELECTROLYTES IN A MAGNETIC FIELD.[1]

BY W. W. STIFLER.

THE fact that most metals when placed in a magnetic field suffer a change in resistance suggests that a similar effect might be produced in liquid conductors. This was first investigated by Neesen [1] and later by Bagard.[2] Quite recently a very exhaustive investigation for mercury, molten bismuth, and various electrolytes was published by Berndt.[3] He used an alternating current method, and fields as high as 3,000 c.g.s. units. His results were negative.

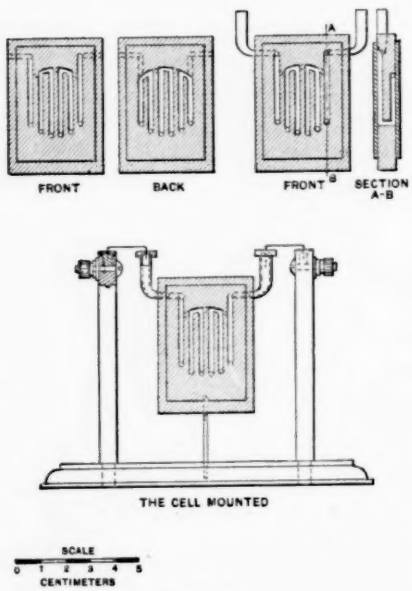

Fig. 1.

In the present investigation, dilute solutions (about 1/100 molal) of cobaltous chloride, ferric chloride, and copper sulphate were tested. The magnetic field was obtained from a large electromagnet of the Du Bois type. The conical pole pieces were about 2.5 cm. in diameter at the smaller end. The average field as measured by a Hartmann and Braun bismuth spiral was about 20,000 c.g.s. units.

The electrolyte was enclosed in a specially constructed cell which permitted a considerable length of the electrolyte to be placed in the field

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, November 28, 1908.

[1] Wied. Ann., v. 23, pp. 482–490, 1884.

[2] Comp. Rend., v. 128, pp. 91–93, 1899.

[3] Ann. d. Phys., v. 23, pp. 932–950, 1907.

without introducing any effects due to induction. It also avoided as far as possible any effects due to the mechanical action of the field on the current. The construction of the cell is shown in Fig. 1. The cell was of hard rubber with glass faces cemented on with de Khotinsky's cement. The grooves were about 2 mm. by 2 mm. in cross-section. The ends of the pole pieces of the magnet entirely covered the central part of the cell. When finished, the cell was about 1 cm. thick. Platinum electrodes were used. These were cleaned with hydrochloric acid, washed with distilled water, and heated to redness before each set of readings.

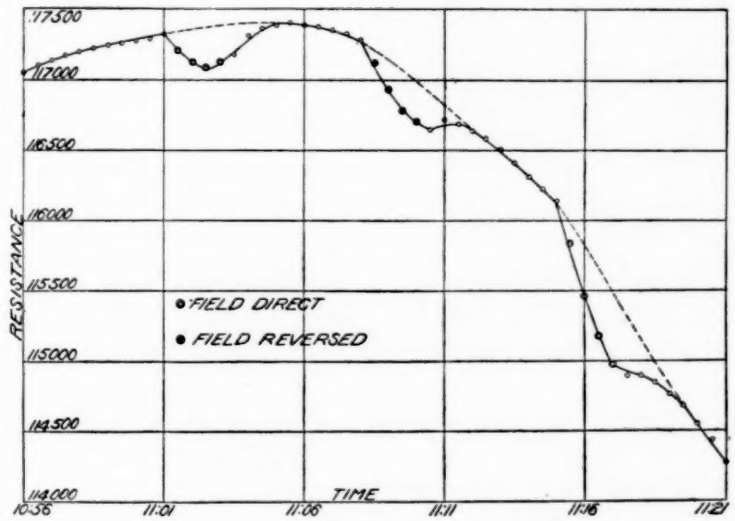

Fig. 2.

The cell when filled with the electrolyte had a resistance of from 100,000 to 500,000 ohms. Owing to this excessively high resistance, it was impossible to use an alternating current method. The cell was connected as one arm of a Wheatstone's bridge of which the ratio arms were 10,000 ohms each. At first a second cell in every way identical with the one just described was used as part of the balancing arm of the bridge. This proved very unsatisfactory, as it was impossible to obtain even an approximate balance. Although an aperiodic galvanometer was used, the index would wander all over the scale in a most erratic manner. However, on substituting resistance boxes for the balancing cells, the galvanometer became very much more steady, though it still showed a tendency to drift steadily in one direction or the other.

Measurements were taken as follows. The battery and galvanometer keys were closed for a few minutes and as exact a balance as possible was

obtained. Then, with both circuits permanently closed, the galvanometer reading was taken at half-minute intervals for five minutes. The magnetic field was then thrown on and the readings were continued for one and one-half or two minutes, after which readings were taken without the field for five minutes, and then again with the field thrown on. A set of readings usually covered from 25 to 45 minutes. The galvanometer was calibrated before and after each set of readings by increasing one arm of the bridge by a known resistance. These results showed that the error introduced by assuming that the change in resistance was proportional to the change in deflection of the galvanometer was within the errors of the method.

From these data the resistance of the electrolyte at half-minute intervals was calculated, and curves were plotted, using time and resistance as coördinates. The general form of these curves, a specimen of which is given in Fig. 2, shows at first an increase in resistance and then a decrease in the resistance of the electrolyte. The former is probably due to polarization; the latter to a slight rise in temperature, although the cell was packed in cotton wool during the readings, and ample time was allowed to insure thermal equilibrium before a set of readings was commenced.

The curves show an abrupt change in resistance when the field is thrown on. This is an average decrease in resistance of approximately

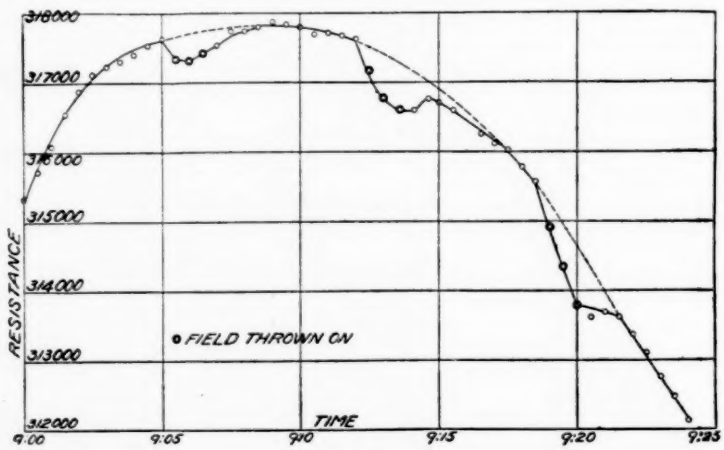

Fig. 3.

0.23 per cent. in cobaltous chloride, and 0.26 per cent. in ferric chloride. This change is independent of the direction of the field and of the direction of the current through the bridge. In copper sulphate the change in resistance is independent of the direction of the field, but is reversed

in sign when the direction of the current through the bridge is reversed. This shows that the change is not a true alteration in resistance but is due to some secondary effect. It seems probable that the effects noticed in cobaltous chloride and ferric chloride were due to some similar cause.

Tests made with the field parallel to the lines of flow showed a change which was reversed in sign in all cases by reversing the direction of the current through the bridge. The fields used in these tests were as high as 3,300 c.g.s. units with the air gap of 7.5 cm.

It is thus concluded that in magnetic fields of strengths up to 20,000 c.g.s. units there is no evidence of any change of electrical resistance in these electrolytes.

This investigation was carried out at the University of Illinois under the general direction of Professor A. P. Carman, to whom the author is indebted for the apparatus and facilities as well as for many suggestions.

PHYSICAL LABORATORY,
UNIVERSITY OF ILLINOIS,
June 1, 1908.

THE LONGITUDINAL PRESSURE OF SOUND WAVES.[1]

BY ELMER E. HALL.

A SIMPLE method of showing the longitudinal pressure of sound waves has been given by Professor Wood.[2] The method, in which the vanes of a light mill-wheel and an electric spark are placed in conjugate foci with reference to a concave mirror, gives, however, not the longitudinal pressure of a train of sound waves but rather that due to a rapid succession of sound pulses.

The longitudinal pressure of a train of sound waves coming from a vibrating tuning fork may readily be shown as a lecture-room experiment. To do this the following simple arrangement is suggested. A torsion balance scheme is used. A finely drawn out glass tube, ten or more centimeters long, has a thin piece of mica two or three centimeters square glued to one end and a compensating piece of soft wax at the other end. A small light mirror attached at the point of balance, with a quartz or single silk fiber attached to the mirror, completes the suspension system. The suspension may be mounted in a box of suitable size, with a tube to hold the fiber, the upper end of the fiber being attached to a short rod passing through a cork thus permitting of easy adjustment. One side of the box should be covered with loosely stretched rubber dam, and another side provided with a glass window. The glass window serves the double

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 27, 1909.

[2] PHYS. REV., Vol. 20, p. 113.

purpose of allowing the passage of a beam of light to and from the reflecting mirror and of enabling one to see when the mica vane is in proper adjustment. The mica vane is set parallel with the side of the box covered with rubber dam, and through a hole in the rubber dam a short heavy brass rod of about the same cross-sectional area as the vane is inserted with the end face parallel with, and one or two centimeters away from, the vane. The other end of the brass rod is outside the box, the rod being held in place by a clamp attached to a ring stand.

If a beam of light be reflected from the mirror and a vibrating tuning fork held firmly against the brass rod a deflection will be obtained showing that the vane has been pushed farther away from the rod. The vibrations communicated longitudinally to the rod are in turn communicated to the air and fall upon the vane. The purpose of the rubber dam is to prevent the material of the box being set in vibration. The writer used a König fork of 256 complete vibrations per second and obtained on a wall 20 feet from the mirror deflections of from 6 to 10 feet. The spot of light does not move for two or three seconds after the vibrating fork is in position, then moves quickly to the position for maximum displacement for this intensity, returning slowly to the zero position as the vibrations die out.

UNIVERSITY OF CALIFORNIA,
DEPARTMENT OF PHYSICS.

CHANGE OF INTENSITY AND RATE OF DECAY OF GAS PHOSPHORESCENCE WITH VARIATION OF GAS PRESSURE.[1]

BY C. C. TROWBRIDGE.

IN a previous abstract[2] the author described a new form of photometer which was specially designed to measure the rate of decay of phosphorescence. Results of measurements on *gas* phosphorescence were presented giving the law of decay. It was shown that the experiments indicated that "afterglows" in gases were cases of true phosphorescence such as is found in solids, and that the law of decay is exactly the same, namely:

$$I = \frac{1}{(a + bt)^2}.$$

In the case of solids, however, the *rate* of decay usually abruptly changes somewhere in the decay curve as first shown by Nichols and Merritt, the decay being slower for long-time than for short-time phosphorescence. This was verified by using the photometer to measure the decay of phosphorescent zinc sulphide.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 27, 1909.

[2] PHYS. REV., June, 1908.

A study of many curves formed by the author of decay of gas phosphorescence seems to show that the rate of decay is the same for a considerable range, that is, b is constant, at least from about one half to one two-hundredth of the original intensity of the phosphorescence. This is due probably to the more simple conditions in a gas than in a solid. Assuming that the rate of decay for very short time phosphorescence is the same as that of the rest of the decay the original intensity of the phosphorescences, or the intensity at the moment that the exciting electrical discharge is cut off, can be determined. In a series of measurements the original intensity I_0 has been thus determined at various gas pressures, all other conditions being approximately constant during the observations. The relation of the original intensity of the gas phosphorescence to the gas pressure was found to be as follows:

$$I_0 = Cp^3.$$

I_0 = original intensity. p = gas pressure.

The original intensity varied from 62 to 4,000. The value of the constant C being $(49.0)^3$, the average variation of C from the mean at six different pressures being less than four per cent., which seems to show that there is no doubt of the validity of this law, at least as an approximation. The gas pressures ranged from 0.08 to 0.32 mm. At one higher pressure the results were too uncertain to be included, but it is about at this pressure, 0.44 mm. where the formation of the glow is also uncertain.

If subsequent variations of the conditions show that there was no constant error due to changes in the gas, etc., during the experiments, then since the pressure determinations depend upon Boyle's law and since the line formed by plotting $\sqrt[3]{I_0}$ and the corresponding gas pressure is not only straight but passes exactly through the origin, and shows no consistent variation between 0.32 to 0.08 mm. gas pressure, there is additional evidence that Boyle's law holds accurately at these low gas pressures. The pressures mentioned are about the range 0.7 mm., where the accuracy of Boyle's law has been questioned by certain experimenters, Bohr, Baly and Ramsay, etc.

The difference in the rate of decay of gas phosphorescence at different pressures has also been determined, if in any case of decay $1/\sqrt{I}$ for each intensity is plotted with the corresponding times a straight line is formed. In the expression $1/\sqrt{I} = a + bt$, b is the slant or tangent which is a measure of the rate of decay. If the different tangents of decays at various gas pressures are plotted with the corresponding gas pressures, a smooth curve is formed which follows closely the form of a parabola, the gas pressure range being from 0.32 down to 0.08 mm. At 0.44 mm. pressure the relation is subject to a decided variation from the simple law of the curve. The decay is rapid at high gas pressures, becomes

slower as the pressure is reduced, is slowest at about 0.26 mm. and gradually becomes rapid again at low pressures.

There have been several possible sources of error in the measurements, for all the factors entering into these experiments have not been understood, but a marked variation from the results given is hardly to be expected. The gas experimented with was nitrogen.

DETERMINATION OF THE FREQUENCY OF GLOW-ARC OSCILLATIONS.[1]

BY W. G. CADY.

IN a former paper[2] a new type of electrical oscillations was described, the characteristic feature of which was shown to be a rapid pulsation between the glow and the arc discharge. Improvements in the apparatus, and the use of a mixture of hydrogen and acetone vapor instead of illuminating gas, have made the oscillations somewhat stronger and more uniform.

The frequency has been determined by exciting, either directly or inductively, a resonating circuit containing a hot wire ammeter. It is shown that the discharge gives rise, simultaneously, to a large number of wave-lengths, suggesting a fundamental vibration with a system of harmonics. This is in agreement with the mathematical theory of circuits containing pulsating resistances.

Both frequency and amplitude of oscillations seem to be sensibly the same with kathodes of silver, copper, aluminium, iron, or lead, though changes in the surfaces of the last two metals soon destroy the effect. Any metal, or carbon, may serve as anode.

The frequency is lowered by decreasing the discharge current, adding a small amount of capacity in the neighborhood of the discharge-tube, or contrary to expectation, by decreasing the self-inductuance by a small amount.

The oscillations are propagated without sensible diminution through several meters of straight wire, but are much damped after passing through a resistance or self-inductance. For this reason, the more remote parts of the circuit are devoid of a trace of the oscillations, giving rise to the apparent paradox that an incandescent lamp placed in series with the arc and close to the latter will glow much more brightly than a similar lamp in a more remote part of circuit. Also, a lamp of high resistance in parallel with any small self-inductance near the discharge-tube will glow, though the resistance of the inductance coil may be negligible. An incandescent lamp, one of whose terminals is held in the hand while the other is touched to any part of the circuit near the arc, lights up.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 27, 1909.

[2] Science, 28, 254, 1908.

Bolometric tests show that the energy radiated from the system can be approximately computed on the assumption that the arc with its adjacent wires behaves like a Hertz oscillator.

Resonance curves were obtained of sufficient accuracy to show that the damping of the oscillations is very small. The amount of energy expended in the discharge, about twenty watts, is, however, too small to make these oscillations applicable to anything beyond the purposes of measurement and demonstration. The greatest amount of energy that has been observed in the oscillating circuit was about fourteen watts.

By the use of a small transformer stepping up to at least 500 volts, quite satisfactory experiments can be performed with an alternating current supply.

Note on Optical Dispersion Formulæ.[1]

By A. Trowbridge.

PROFESSOR R. C. MacLaurin has recently shown[2] that it is possible to so determine the constants of the dispersion formula proposed by him that it not only represents adequately the dispersion of rock-salt and sylvite throughout the entire visible and infra-red spectrum but also predicts the positions of the absorption bands in these substances more nearly where they are found by direct experiment to lie than does the Sellmeier type of dispersion formula. Moreover there is better agreement between the observed and calculated values of the dielectric constant

There are theoretical reasons[3] for assuming that one of the constants of the MacLaurin formula should have the value 2 in the case of a fluid.

The present writer has determined the constants of the formula so that it represents the dispersion of carbon bisulphide over the range studied by Rubens,[4] *i. e.*, from 0.434 μ to 1.998 μ.

In order that the observed and calculated values may agree to within one part in one thousand it is necessary to assume at least one infra-red absorption band. The calculated position of this band is $\lambda = 8\,\mu$ and that of the ultra-violet band $\lambda = 0.2244\,\mu$.

The writer has endeavored to find the infra-red band by the method of "Rest Strahlen" in conjunction with a spectro-bolometer of adequate sensibility. While the energy of wave-length $\lambda = 8\,\mu \pm \delta\lambda$ in the incident beam was sufficient to give a steady galvanometer deflection of 100

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 27, 1909.

[2] Proc. Roy. Soc. A, Vol. 81, No. A 548.

[3] Cf. Larmor, Phil. Trans., 1897 A, p. 233.

[4] Wied. Ann., 45, 1892.

scale divisions, two reflections from a surface of carbon bisulphide practically reduced the deflection to zero.

The entire spectral region between the red and $\lambda = 8.5\,\mu$ was investigated with the result that no trace of metallic reflection of the liquid was found.

It would thus seem that the MacLaurin formula in the special form proposed by Larmor for fluids

$$\frac{n^2 - 1}{n^2 + a} = \frac{k - 1}{k + a} + \frac{c_1}{\lambda^2 - \lambda_1^2} - \cdots - \frac{c_n}{\lambda^2 - \lambda_n^2}$$

where $a = 2$ does not hold.

If a be retained as an *arbitrary* constant as in the MacLaurin formula for solids then it is possible to determine the constants to give calculated values of the index of refraction agreeing with observation without assuming the existence of a band in the nearer infra-red spectrum.

NEW BOOKS.

A Text-Book of Physics. Edited by A. WILMER DUFF. Philadelphia, P. Blakiston's Son & Co. Pp. xi + 680.

This text-book is the result of collaboration of several winters. The different divisions of the work are treated by men who are teaching physicists and who are specialists in the divisions of the subject treated. Professor A. Wilmer Duff, who has edited the work, presents the part of the work devoted to mechanics and the properties of matter in the same clear manner as he has presented the subject of mechanics in his laboratory manual. That portion of the book devoted to heat is written by Professor Guthe, of the State University of Iowa. With a little extension of the discussion of thermometry, radiation and thermodynamics and with a discussion of the kinetic theory of gases added this part of the book might find a field for itself. There is a growing demand among teachers for separate books on the various phases of the work in physics in order that they may use such material as may seem best suited to their individual needs. The present text is to a certain extent a recognition of this demand.

Wave motion and Sound are discussed by Professor William Hallock, of Columbia; Light, by Professor E. Percival Lewis, of the University of California. Electricity and Magnetism, by Professor Arthur W. Goodspeed, of the University of Pennsylvania; Electromagnetic Induction, by Professor A. P. Carman, of the University of Illinois; and Conduction of Electricity through Gases and Radio-activity, by Professor R. K. McClung, of Mount Allison University, Sackville, N. B.

In the part of the work on electricity and magnetism a more complete discussion of Ohm's and Kirchhoff's laws would have added to that section. The tangent galvanometer theory is discussed but as usual the d'Arsonval galvanometer is passed by without the careful discussion of its theory that it merits in the light of its general usefulness and the fact that it is to a great extent replacing the tangent galvanometer in everyday use.

Lists of problems are included at the ends of various sections. These lists might be extended to advantage if any lists are to be included at all. A large and well selected list of problems enables a saving of time in the class room in mechanical work and permits of variation of assignments from year to year where a book is in continuous use.

In mechanical make-up the book is not as good as the subject-matter deserves. The pages have a solid look, due to the type probably, some of which is much too small to be easily read. Many of the figures are small, in some the lines are not sharp and the lettering or numbering is indistinct.

ERNEST BLAKER.

Physikalische Freihandversuche. By H. HAHN. Part. II., Eigenschaften der Flüssigkeiten und Gase. 8vo, pp, vii + 293. Berlin, Otto Salle, 1907.

Technik des physikalischen Unterrichts. By F. C. G. MÜLLER. 8vo, xii + 370. Berlin, Otto Salle, 1906.

Hahn's "Freihandversuche," based in part upon the notes of the late Professor B. Schwalbe, of the Dorotheenstädisches Realgymnasium of Berlin, is devoted to the description of illustrative experiments requiring the absolute minimum of apparatus. The book is a most unusual one in its interest and suggestiveness. Part II. of the work, which is now before us, deals chiefly with the properties of liquids and gases, and contains brief descriptions of 674 experiments. The apparatus required consists of tumblers, corks, test-tubes, pieces of wire, matches, and the like, and of *simple* combinations of these; a bicycle pump is almost the most elaborate piece of apparatus mentioned. The manipulation required is also simple in the extreme. After an inspection of this book it seems hardly possible that any teacher of physics can complain that the concrete illustration of his subject is not practicable on account of insufficient equipment.

The experiments are arranged systematically and are so well illustrated by simple diagrams that it is often unnecessary to read the text. While in many instances it may prove desirable to perform the experiments exactly as described, I think that in a majority of cases the book will prove most valuable in suggesting experiments with other apparatus. For example in the case of surface tension the experiments are suitable for small classes or for individual work by the student, but could not be easily shown to a large class. In such cases the many valuable suggestions may be utilized in devising experiments suited to whatever projecting apparatus may be available. The book is the most satisfactory one of its class that I have seen and will be found useful by every teacher of elementary physics, whether his illustrative equipment is large or small.

Müller's "Technik des physikalischen Unterrichts" deals more especially with elementary laboratory work. The apparatus called for is also simple, but more elaborate than that required for the "Freihandversuche." The book contains many useful hints, but corresponding as it does to the requirements of the German Gymnasium will be less generally useful in this country.

E .M.